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ACTIVE RC NETWORK SYNTHESIS WITH OPERATIONAL AMPLIFIER

by

Erol Yuksel

United States
Naval Postgraduate School

THESIS

ACTIVE RC NETWORK SYNTHESIS WITH OPERATIONAL AMPLIFIER

by

Erol Yuksel

September 1970

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Active RC Network Synthesis with Operational Amplifier

by

Erol Yuksel
Lieutenant, Turkish Navy
Turkish Naval Academy
B.S., Naval Postgraduate School, 1969

Submitted in partial fulfillment of the
requirements for the degree of

ELECTRICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL
September 1970

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Y, CALIF. 93940

ABSTRACT

Active RC network synthesis techniques with operational amplifiers are reviewed, discussed, and classified according to the number of amplifiers and number of feedback paths in the circuit. After presenting the main properties of active RC network theory, various synthesis techniques are discussed and evaluated according to their merits, by means of sensitivity and stability theory. A modification to Lovering's circuit is proposed.

Six design examples are presented to illustrate the application of the techniques and to observe the effect of nonideal active and passive components. The designs are practically realized, their performance is tested and experimental results are presented. Reasons for the discrepancies found between theory and experimental results are discussed.

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I. INTRODUCTION

A. HISTORICAL BACKGROUND

Perhaps the first application of active RC networks occurred in 1931 when Crisson [1] at the Bell Telephone Laboratories built an active repeater, which functioned to make up losses in telephone lines. Scott [2] in 1938 discussed the use of RC networks in the realization of low-frequency selective circuits. At low frequencies industors became impractical because of their large size and it is impossible to achieve a satisfactory Q. Scott described an RC twin-T circuit in the feedback loop of a high gain amplifier which he used to design his wave analyzer. However the fundamentals of feedback amplifier theory had been elucidated and summarized by Bode [3] in 1945, and they have remained the basis of all subsequent work on feedback, stability and sensitivity.

Shortly after Bode's work, Tellegen [4] in 1948 completed the class of passive network elements by his introduction of the gyrator. However the construction of an ideal passive gyrator has not, up to this time, 1970, proven successful, and many workers have been concerned with the construction of ideal active gyrators. Such gyrators are built with active elements and either have equal or unequal gyration immittances. The first of these gyrators were built by Bogert [5] and Sharpe [6], and realized with vacuum tubes. Later in 1965 Shenoi [7] built a transistorized gyrator and advanced a method of active RC network synthesis using gyrators.

The invention of transistors in 1948 also made possible a number of useful and interesting active elements. One of these, the negative impedance converter (NIC), was described by Merril [8] in 1951. Shortly after, Linvill [9, 10] in 1953 designed an active RC filter using negative impedance converters. Since then a great deal of study of active RC circuits has appeared in the literature.

There are numerous reasons for the interest in active RC circuits and filters, some of which will now be mentioned. Ideal passive RLC filters require the use of inductors and these, as has been mentioned before, are difficult or impossible to realize for low frequency use. Specifically, one requires a high quality factor, defined by

$$Q = \frac{\omega L}{R_L} \tag{I.1}$$

where ω is the angular frequency, L the inductance, and R_L the positive resistance of the winding. Practical inductors are subject to stray capacitance, suffer from skin effect at high frequencies and also depend on the core material used, and also on several other factors. In practice, losses increase with frequency and Q becomes a complex function of frequency. In the low-frequency range practical inductors with reasonable Q become bulky and expensive. On the other hand, at high frequencies an inductor may look like a capacitor due to parasitics. Practically, there is a lower limit and an upper limit in the frequency spectrum in which inductors with good Q's and reasonable L's can be built. A practical curve of the useful inductance range is shown in Fig.

I.1. As may be seen from Fig. I.1, crystal filters are useful in a somewhat higher but restricted frequency range. However, in the low-frequency range no practical realizable inductors exist, and it is this deficience which active RC filter synthesis has attempted to solve in the first instance.

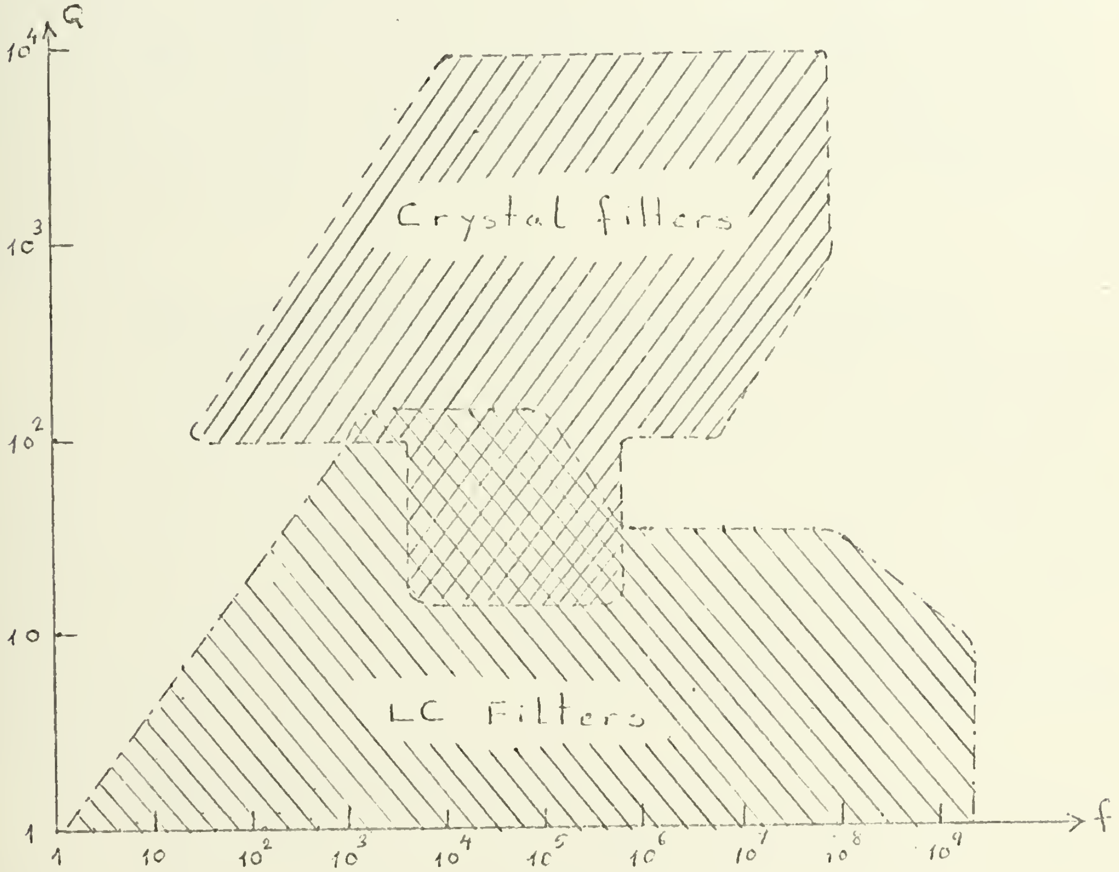


Figure I.1 Useful Inductance Range in Frequency Spectrum [59].

Recently the appearance of integrated circuits with their small size, lightweight, low cost, low power consumption and high reliability have made the construction of active RC networks attractive and practical. RC active synthesis appears to have pursued several directions.

1. Use of Operational Amplifiers

The operational amplifier as a building block offers certain advantages to the circuit designer since good operational amplifiers are readily available in integrated form. Their power consumption is modest, they are reliable, robust and cheap. Present disadvantages are their limited frequency range, which sets a practical upper limit to their usefulness in RC active synthesis. Every RC active device can be constructed in terms of only R's, C's and operational amplifiers.

2. Use of Standard Second Order Filter Sections

For many RC active realizations, certain combinations of second-order filter sections may be used to produce the desired characteristics. The second-order filter section as a building block has been discussed by Moschytz [11], Kerwin, Huelsman, Newcomb [12] and others and networks have been constructed with excellent characteristics and sensitivities.

3. Simulation of Inductors

Industors may be constructed in terms of R, C and active elements. Orchard [13] has pointed out that the resistively terminated LC ladder filters have low sensitivity to passive element variations. Consider for example the second-order low pass ladder of Fig. I.2. The transfer impedance of this filter is

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} = \frac{\frac{R}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

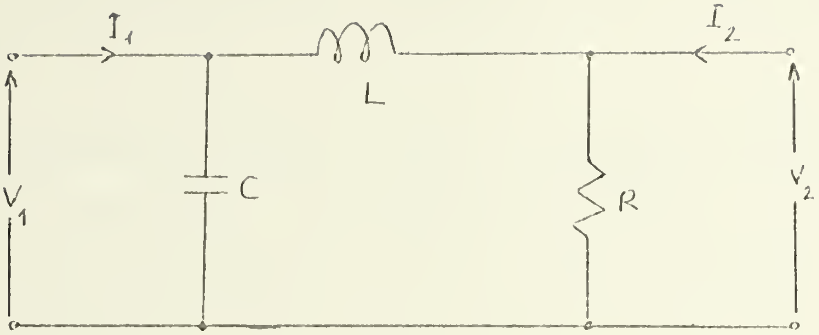


Figure I.2 A Second Order Low-Pass Filter.

Consider now the denominator. This may be written as

$$s^2 + 2\sigma s + (\sigma^2 + \omega_c^2)$$

where $2\sigma = R/L$ and $(\sigma^2 + \omega_c^2) = 1/LC$

The Q is now defined as $\omega_c / 2\sigma$ and if

$$\sigma << \omega_c$$

the Q of the pole pair is

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{I.2}$$

The sensitivity of Q with respect to variations in a network element E is

$$S_E^Q = \frac{dQ/Q}{dE/E} \tag{I.3}$$

Using Eqs. I.2 and I.3 for R, L and C sensitivities are obtained

$$S_R^Q = -1 \quad , \quad S_L^Q = 1/2 \quad , \quad S_C^Q = -1/2$$

These sensitivities are low and are independent of Q. Thus a simple way of designing a good inductorless filter is to design an LC filter first and then replace the inductors by simulated inductors. Provided the simulated inductors have

good Q's and good Q sensitivities a useful filter will be obtained.

B. SOME CONSIDERATIONS ON ACTIVE RC NETWORK THEORY

Before the advent of integrated circuits, one of the main concerns of circuit design was to reduce the number of active elements as much as possible because of size cost, power and reliability considerations. However integrated circuits have eliminated this need because of the ease with which large numbers of transistors can be constructed on one die. Nevertheless, some criterion or figure of merit which can be applied to the many synthesis techniques that have been proposed is required. One criterion is the stability of the network to oscillation. Another useful criterion is the quality factor Q, and the sensitivity of the system function to active and passive parameter variations. These topics are now briefly discussed.

1. Stability

A major problem in the design of an active RC network is stability. A passive RLC circuit can never become unstable with change of element values, because all its poles will always lie in the left half s-plane. An active network on the other hand, can become unstable with a slight variation of either a passive or active circuit parameter, because poles in the left half s-plane may then be shifted into the right half s-plane.

The necessary and sufficient conditions for stability of a linear lumped finite (LLF) network can be stated as follows [14]:

a. A LLF network under a given mode of operation is strictly stable if the corresponding network function has no poles in the right half s-plane including the jω axis.

b. A LLF network under a given mode of operation is stable or marginally stable if and only if the corresponding network function has poles in the left half s-plane and simple jω axis poles (if any).

2. Quality Factor Q

The quality factor Q of a passive inductor was given in Eq. I.1. Now consider an inductor simulated by a gyrator loaded with a capacitor as shown in Fig. I.3.

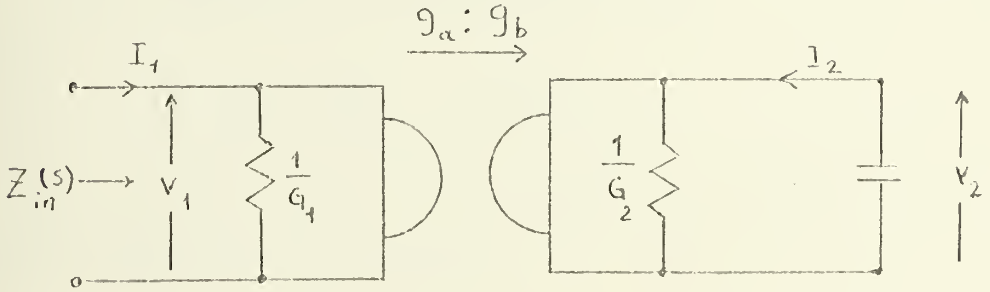


Figure I.3 Simulated Inductance by Gyrator.

The nonideal gyrator is characterized by its two port admittance matrix

$$\underset{\sim}{Y} = \begin{bmatrix} G_1 & g_a \\ -g_b & G_2 \end{bmatrix} \qquad (I.4)$$

where G_1 and G_2 are small numbers representing the input and output resistive losses, and g_a and g_b represents forward and backward gyration admittances. From Fig. I.3

$$V_2 = -I_2/Cs \qquad (I.5)$$

The input impedance of the loaded gyrator is obtained from Eqs. I.4 and I.5,

$$Z_{in}(s) = \frac{G_2 + Cs}{G_1 G_2 + g_a g_b + G_1 Cs} \tag{I.6}$$

Substituting s = jω in the Eq. I.6 and specifying real and imaginary parts R_{eq} and L_{eq} yields

$$R_{eq} = \frac{G_1 G_2^2 + G_2 g_a g_b + \omega^2 C^2 G_1}{(G_1 G_2 + g_a g_b)^2 + \omega^2 C^2 G_1^2} \tag{I.7a}$$

$$L_{eq} = \frac{C g_a g_b}{(G_1 G_2 + g_a g_b)^2 + \omega^2 C^2 G_1^2} \tag{I.7b}$$

and so

$$Z_{in}(s) = R_{eq} + j\omega L_{eq} \tag{I.8}$$

As may be seen from Eq. I.8, $Z_{in}(j\omega)$ is identical to a lossy inductor where Q is given by

$$Q = \frac{\omega L_{eq}}{R_{eq}} = \frac{\omega C g_a g_b}{G_1 G_2^2 + G_2 g_a g_b + \omega^2 C^2 G_1}$$

Note that Q is a complex function of frequency ω which has a maximum Q value

$$Q_{max} = \frac{g_a g_b}{2(g_a g_b + G_1 G_2)} \sqrt{1 + \frac{g_a g_b}{G_1 G_2}} \tag{I.9}$$

at

$$\omega = \frac{G_2}{C} \sqrt{1 + \frac{g_a g_b}{G_1 G_2}}$$

Note also that as ω becomes very large Q will approach zero.

It is also possible to treat the loaded gyrator of Fig. I.3 in a slightly different manner. The loaded gyrator with

a capacitor of value C_1 across the input is considered as a resonant circuit, shown in Fig. I.4

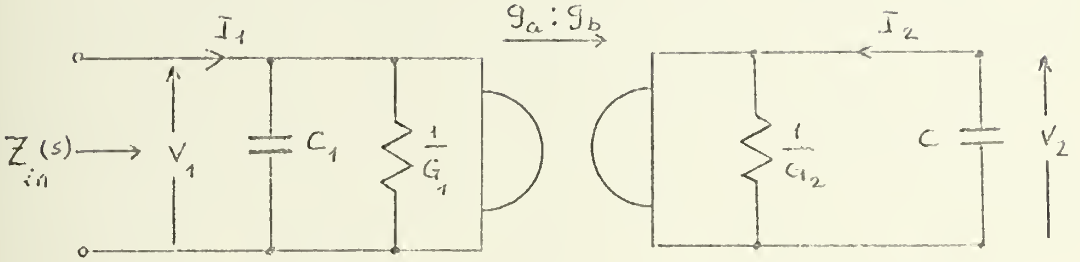


Figure I.4 A Resonance Circuit with Simulated Inductance.

The input impedance

$$Z_{in}(s) = \frac{\frac{1}{C_1} s + \frac{R_{eq}}{C_1 L_{eq}}}{s^2 + \frac{R_{eq}}{L_{eq}} s + \frac{1}{C_1 L_{eq}}}$$

and its equivalent circuit is shown in Fig. I.5.

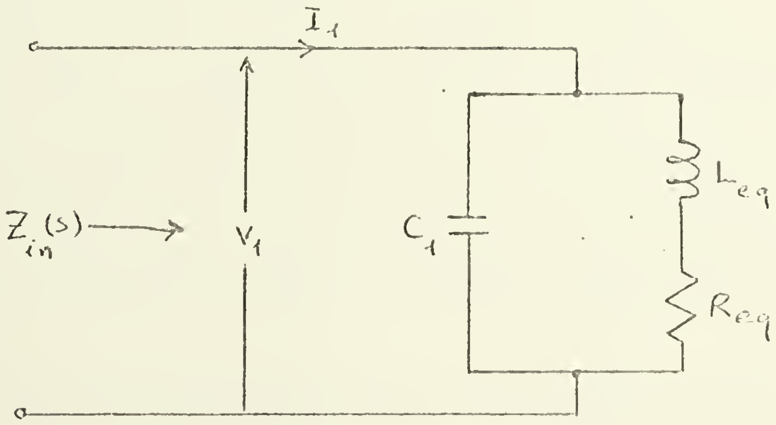


Figure I.5 Equivalent Circuit of Gyrator Resonant Circuit

The maximum resonant Q of this connection can be shown to be [15].

$$Q_{max} = \sqrt{\frac{g_a g_b}{G_1 G_2}} \tag{I.10}$$

Thus the maximum or resonant Q of any active RC one port network which simulates an inductor is seen to be a function of the elements of the gyrator two port matrix in Eq. I.4. Equations I.9 and I.10 show that to obtain high Q inductors and high resonance Q, the gyrator must be designed with values of G_1 and G_2 that are very small with respect to the gyration admittances. Therefore if high Q is desired, the active RC synthesis which realizes the circuit with highest Q is preferred. Sensitivity of Q with respect to active and passive network parameters is also an important factor in selecting a specific synthesis technique. Q may be defined also with respect to a second-order section as follows: Consider the bandpass transfer function

$$T(s) = \frac{Ks}{s^2 + 2\sigma s + (\sigma^2 + \omega_c^2)} \tag{I.11}$$

where K is the gain factor of the transfer function, σ and ω_c are the real part and the magnitude of the imaginary part of the pole pair. The locations of the zero and pole pair of T(s) is shown in Fig. I.6. ω_n is defined as the magnitude of the vector drawn from the origin to one of the poles. Hence $\omega_n^2 = \sigma^2 + \omega_c^2$.

The frequency response of T(s) is shown in Fig. I.7 The sharpness of the peak of the response is defined as the ratio of the resonant frequency ω_n to the half power band width where

$$BW_{3db} = (\omega_2 - \omega_1) = 2\sigma$$

For very small band widths

$$\sigma << \omega_c$$

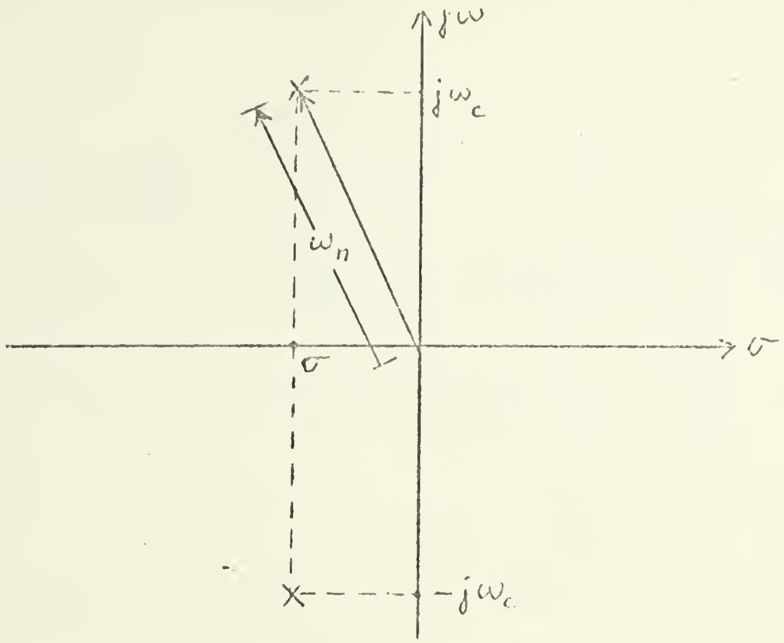


Figure I.6 The Pole-Zero Diagram of a Second-Order System.

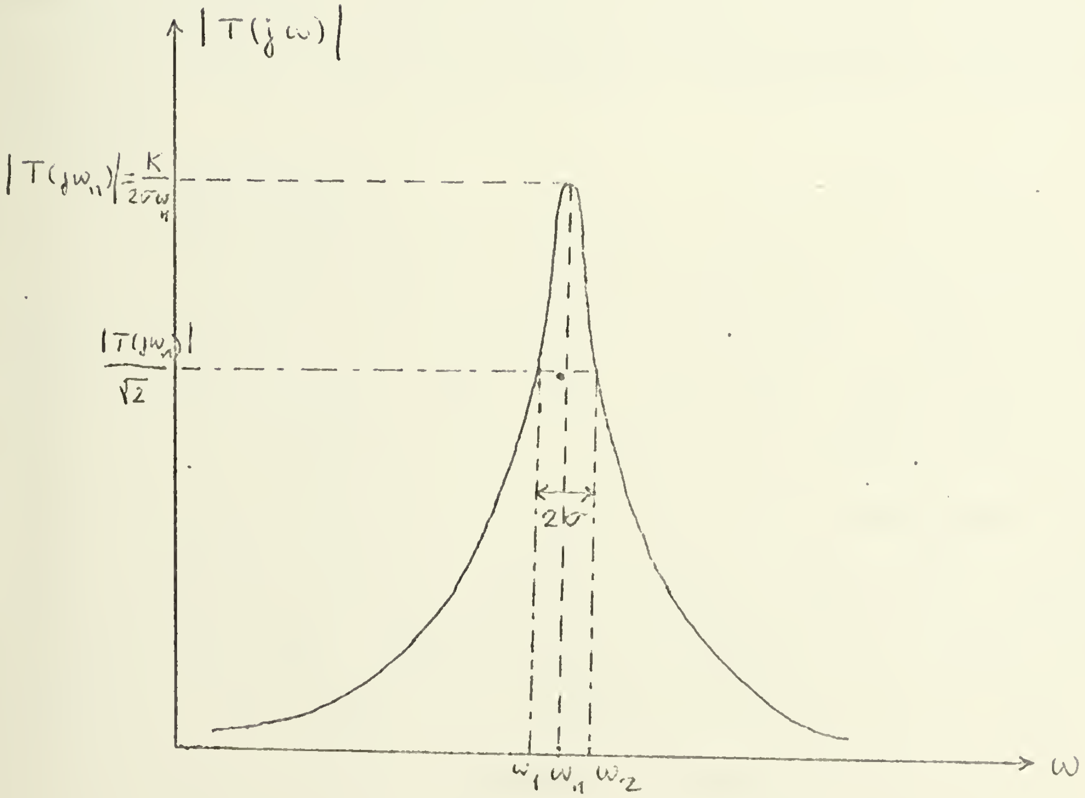


Figure I.7 The Frequency Response of Second Order Section.

hence $\omega_n \simeq \omega_c$

also $Q = \frac{\omega_c}{2\sigma} \simeq \frac{\omega_n}{2\sigma}$

Therefore for very small bandwidths the sharpness of the peak can be defined as Q.

As may be seen from Fig. I.6 the quality factor determines the location of the poles. If Q is high $\sigma \ll \omega_c$ and this implies that these poles are located very near the jω axis. If the location of poles are sensitive to the variations of network parameters, there is a possibility that the poles may drift into the right half plane. Then the circuit becomes unstable. Therefore a synthesis method which gives the least pole sensitivity (least Q sensitivity) should be selected.

3. Sensitivity

Characteristics of active devices and parameters of passive components are subject to change for various reasons such as temperature, bias level, humidity, aging and so on. The point of concern here is the effect of these variations on the characteristics of the circuit to be designed. Specifically parameter variations will effect pole zero locations, Q, gain and phase. Sensitivity is defined as a quantitative measure of the change in a network characteristics due to a change in the network parameter. Usually there are six sensitivities of interest. These sensitivities are:

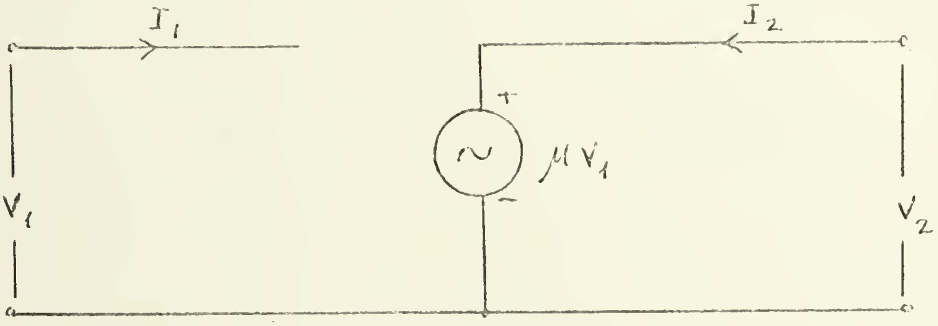
a. Gain sensitivity
b. Phase sensitivity
c. Polynomial sensitivity

d. Pole (zero) sensitivity
e. Coefficient sensitivity
f. Q sensitivity

Since sensitivity is an important criterion in the design of active RC networks, this subject will be discussed in more detail in the second chapter.

C. ACTIVE ELEMENTS

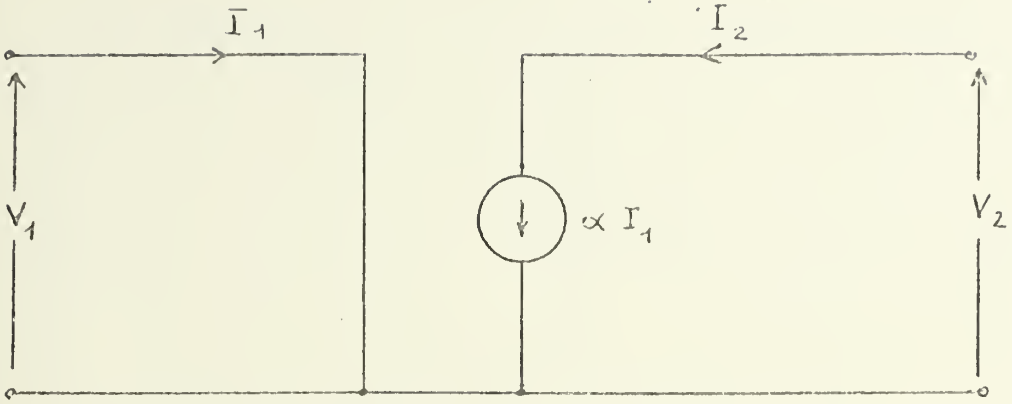
The commonly used active elements for active RC synthesis are negative impedance converter, the controlled sources, the gyrator, and the operational amplifier. The circuit symbols and the idealized characteristics of these active elements are shown in Figs. I.8 through I.14.



$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \mu & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} , \left(\frac{dV_1}{dI_1} \to \infty ; \quad \frac{dV_2}{dI_2} \to 0 \right)$$

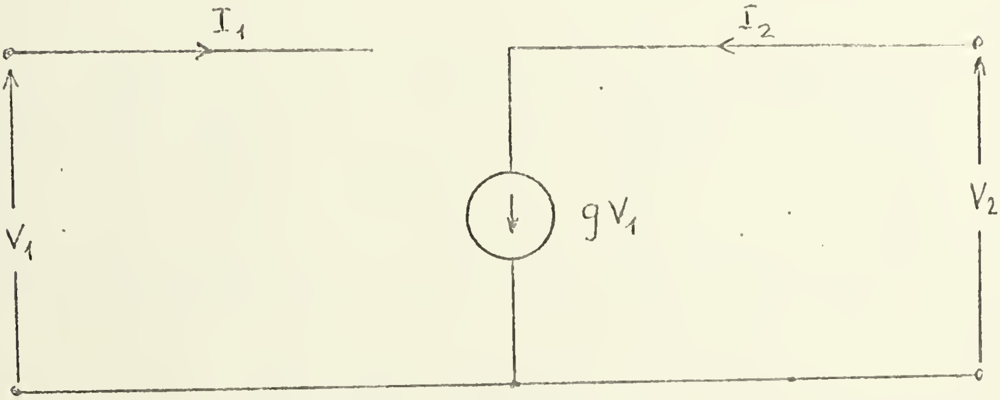
Figure I.8 The Voltage Controlled Voltage Source.

In this study, the operational amplifier has been selected as the active element because of its easy availability in integrated circuit form and its reliability over a wide range of frequency. Operational amplifiers find many applications,



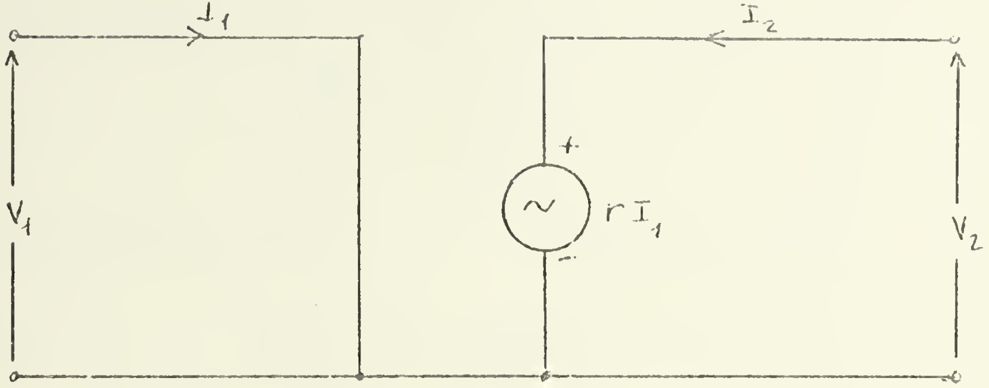
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} , \quad \left(\frac{dV_1}{dI_1} \to 0 \; ; \; \frac{dV_2}{dI_2} \to \infty \right)$$

Figure I.9 The Current Controlled Current Source



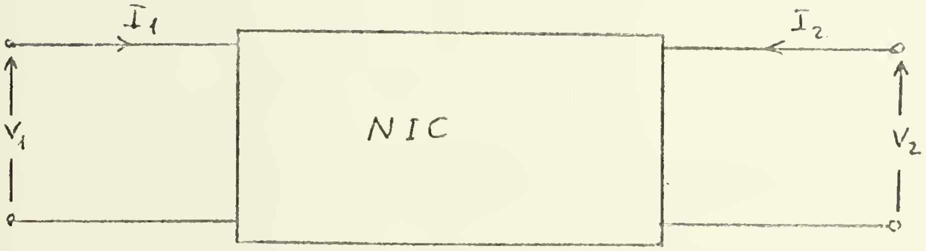
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ g & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} , \quad \left(\frac{dV_1}{dI_1} \to \infty \; ; \; \frac{dV_2}{dI_2} \to \infty \right)$$

Figure I.10 The Voltage Controlled Current Source.



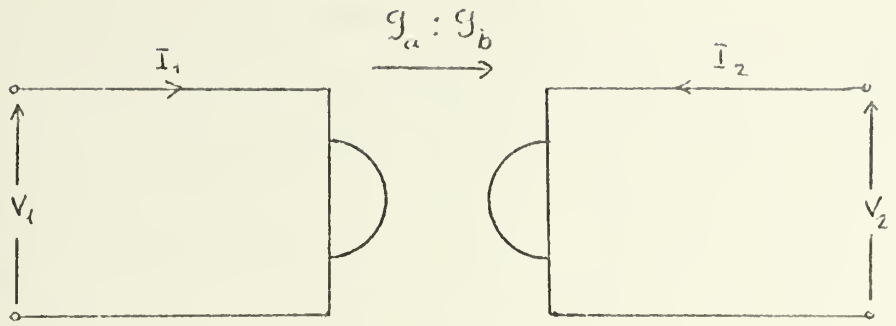
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ r & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \quad \left(\frac{dV_1}{dI_1} \to 0 \; ; \; \frac{dV_2}{dI_2} \to 0 \right) .$$

Figure I.11 The Current Controlled Voltage Source



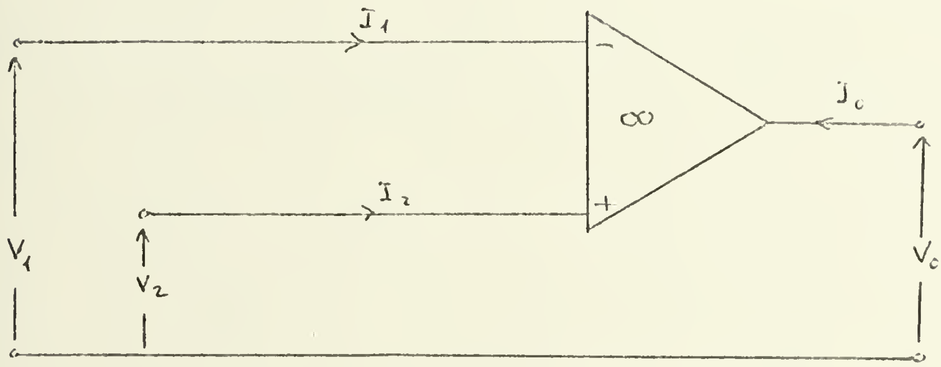
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & k_1 \\ k_2 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Figure I.12 The Negative Impedance Converter



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 & g_a \\ -g_b & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Figure I.13 The Active Gyrator.



$$V_o = A(V_2 - V_1)$$

$$A \rightarrow \infty \qquad \left(\frac{dV_1}{dI_1} \rightarrow \infty \; ; \; \frac{dV_2}{dI_2} \rightarrow \infty \; ; \; \frac{dV_o}{dI_o} \rightarrow 0 \right)$$

$$I_1 = I_2 = 0$$

Figure I.14 The Operational Amplifier

particularly in the areas of control systems, communications and analog computers. Because of their very low output impedances (ideally zero), these amplifiers do not create any matching problem for the load circuits. All of the active elements described before can be easily implemented in terms of operational amplifiers.

Given in Fig. I.14 is the transfer characteristic. The gain A is assumed to be independent of the frequency, temperature, and the input voltage levels, and is assumed to approach infinity. Hence the input impedances are infinite, and the output impedance is zero. However, the characteristics of a practical amplifier deviate from the ideal. For example, gain is not infinite but begins from a high value at DC, and decreases almost monotonically with the frequency. A typical open loop frequency response curve of an operational amplifier is shown in Fig. II.15, and is discussed further in section II.E. The phase characteristic is also a function of frequency. The input and output impedances are not infinite and zero respectively, both have finite values. Input currents are not zero and are not equal to each other; their difference is the input offset current. Output voltage is not only a function of the difference of the voltages at the two inputs (differential gain), but is also a function of the magnitude of the input voltages (common mode gain) and the supply voltages. There is a dynamic range in the input and output voltage swings which if exceeded causes input and output to be nonlinearly related. Hence there is quite a difference

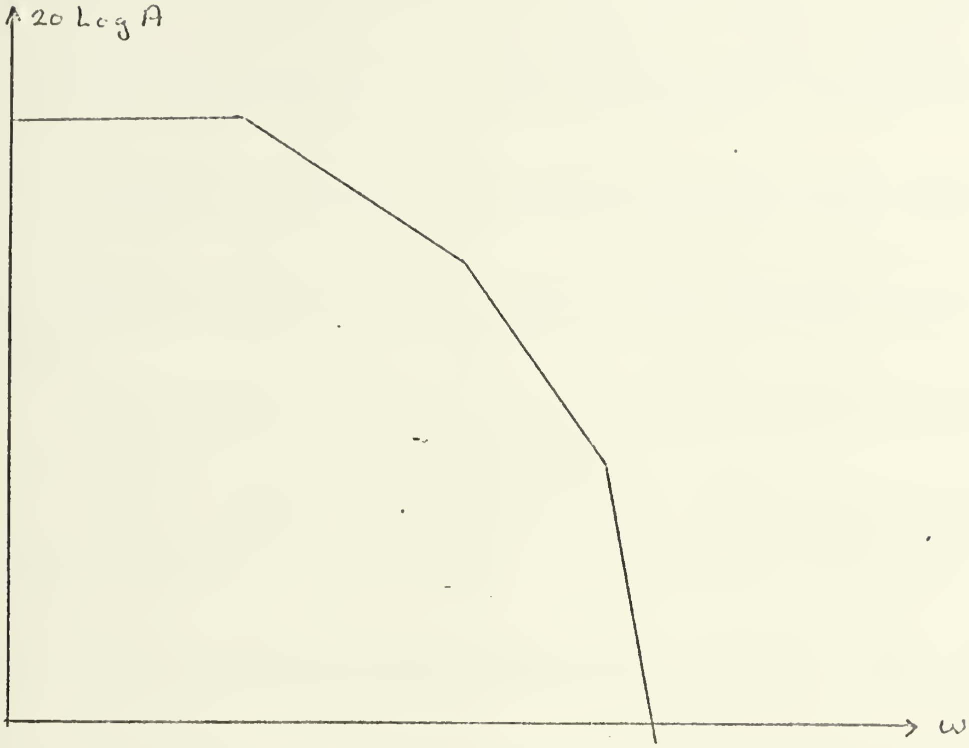


Figure II.15 The Open Loop Frequency Response of an Operational Amplifier

between the practical operational amplifier and its idealized model. The characteristics of the non-ideal operational amplifiers are usually supplied by the manufacturer in terms of the definitions given below.

1. Input offset voltage: The voltage which must be applied between the input terminals to obtain zero output.

2. Input offset current: The difference in the currents into the two input terminals with the output at zero volts.

3. Input bias current: The average of the two input currents.

4. Input resistance: The resistance looking into either input terminal with the other grounded.

5. Input capacitance: The capacitance looking into either input terminal with the other grounded.

6. Large-signal voltage gain: The ratio of the maximum output voltage swing to the change in input voltage required to drive the output from zero to this voltage.

7. Output impedance: The resistance seen looking into the output terminal with the output at null, i.e., the output is zero with zero volts input.

8. Transient response: The closed-loop step function response of the amplifier under small signal conditions.

9. Input voltage range: The range of voltage which, if exceeded on either input terminal, could damage the amplifier.

10. Common mode rejection ratio: The ratio of differential mode gain to common mode gain.

11. Supply voltage rejection ratio: The rate of the change in input offset voltage to the change in supply voltage producing it.

12. Output voltage swing: The peak output swing, referred to zero that can be obtained without clipping.

13. Slew rate: Maximum rate of change of output voltage for a large input step change (measured in volts per microsecond). If these parameters are known it is possible to approach ideal amplifier performance for a specific design in a restricted frequency range.

II. ON THE SYNTHESIS OF RC-OPERATIONAL AMPLIFIERS

A. REVIEW OF PROPERTIES OF RC NETWORKS

Some of the properties of the different classes of RC networks are briefly reviewed as follows:

1. Passive RC Networks

Passive RC networks are considered as the positive resistance and positive capacitance (+R, +C) class of network. As is well known, the natural frequencies of driving functions are restricted to the negative real axis of the s-plane as shown in Fig. II.1a.

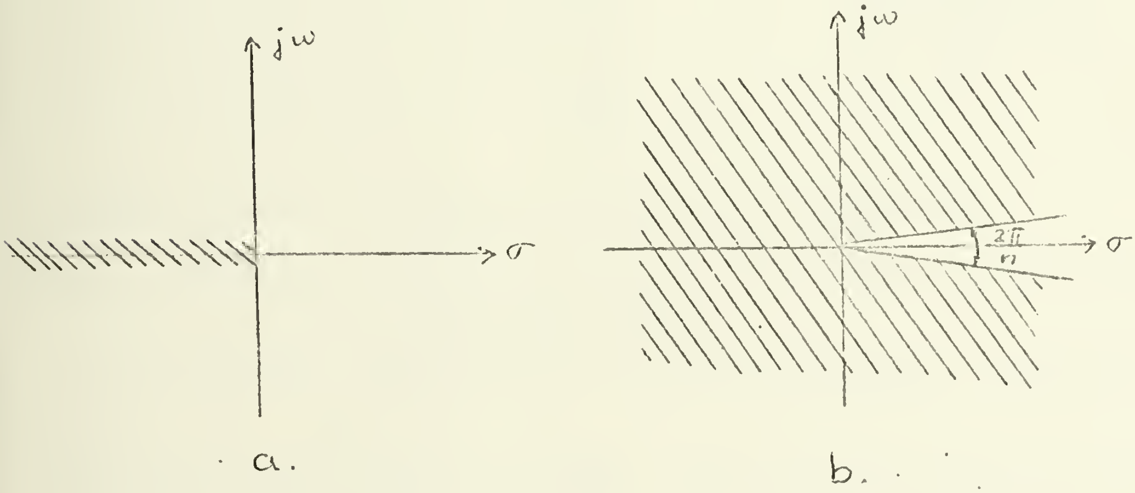


Figure II.1a - Location of Natural Frequencies.

" II.1b - Locations of zeros of transfer functions of 3-Terminal Grounded Networks.

Poles and zeros must be simple, however zeros of transfer functions may be located anywhere in the s-plane. Zeros of 3-terminal network transfer functions may be located anywhere in the s-plane except the positive real axis and on a wedge

of angle 2π/n radian surrounding it, where n is the degree of the numerator polynomial. This is shown in Fig. II.1b.

2. Active RC Networks, ± R, +C Networks [16,17]

The stable natural frequencies must be located on the negative real axis as shown in Fig. II.2a.

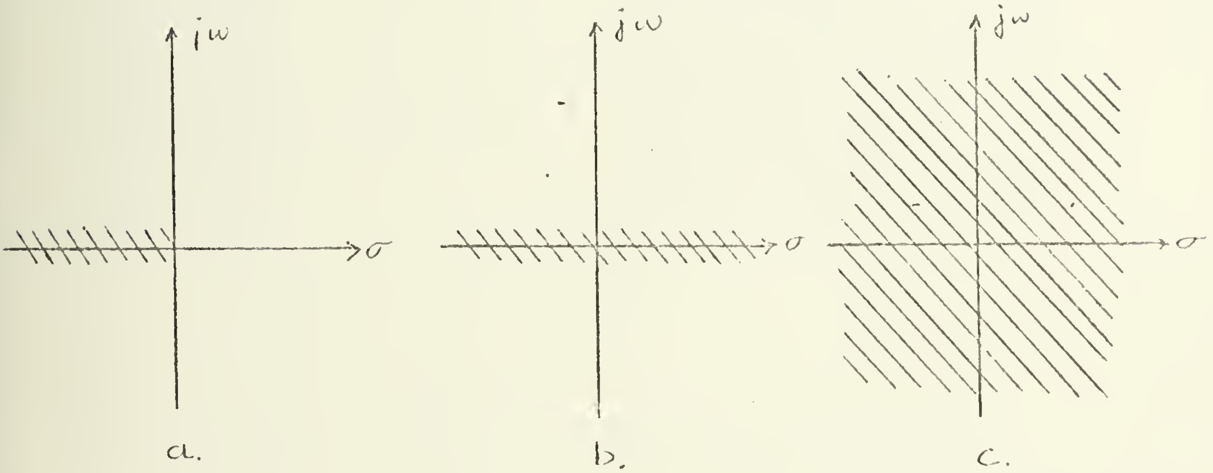


Figure II.2a - Locations of Natural Frequencies.

" II.2b - Locations of Poles and Zeros of Driving Point Functions.

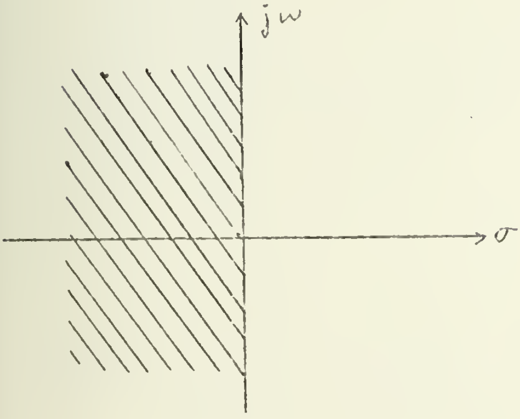
" II.3c - Locations of Zeros of Transfer Function.

Poles and zeros must be simple; zeros of driving point functions may be anywhere on the real axis as shown in Fig. II.2b. Zeros of transfer functions may be located anywhere in the s-plane as shown in Fig. II.2c. This class of network is capable of voltage gain over the entire frequency range although Fig. II.2 differs relatively little from the Fig. II.1.

3. Passive RC and Gyrator Class of Networks [4, 18, 19].

It is well known that a capacitively loaded gyrator simulates at its inputs an inductor. Hence the RC-gyrator

class of networks must at least be as general as the RLC class of networks. Hence natural frequencies may be located anywhere in the left half of the s-plane including the jω axis as shown in Fig. II.3a.



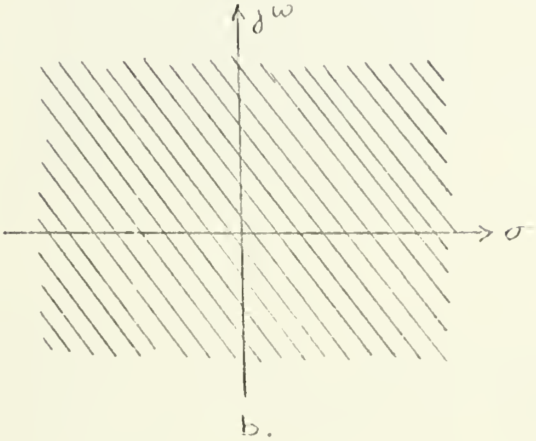


Figure II.3a - Locations of Natural Frequencies.

" II.3b - Locations of Zeros of Transfer Functions.

Poles and zeros on the jω axis must be simple. Zeros of transfer functions may occur anywhere on the s-plane as shown in Fig. II.3b. Because the gyrator is a nonreciprocal network element this class of network is capable of nonreciprocal behavior.

4. Passive RC-NIC or Operational Amplifier Class of Networks [20, 21, 22].

The stable natural frequencies must be located in the left half s-plane including the jω axis as shown in Fig. II.4a. Natural frequencies on the jω axis must be simple. Zeros of driving point functions and zeros, of transfer functions may be located anywhere as shown in Fig. II.4b. These

properties are also shared by the passive RC controlled source class of networks.

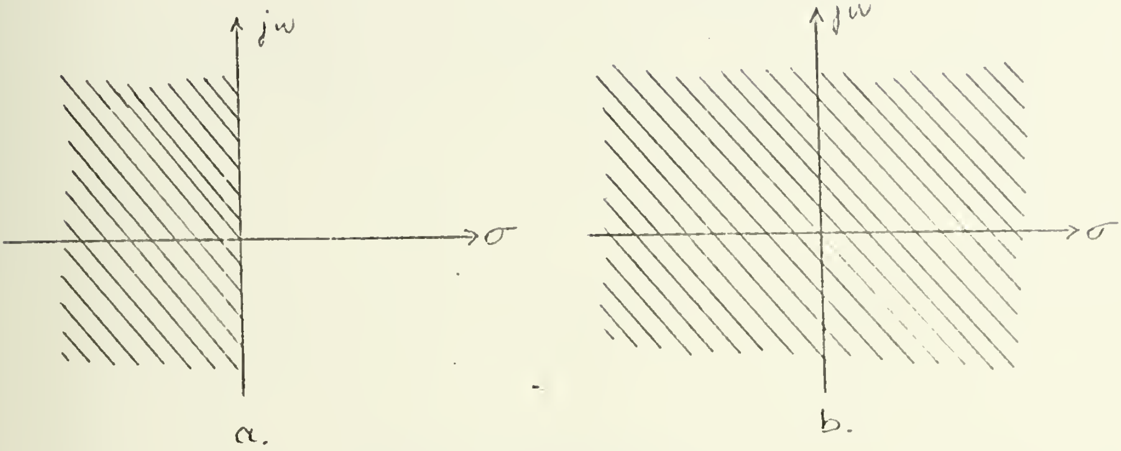


Figure II.4a - Locations of Stable Natural Frequencies.

" II.4b - Locations of Zeros of Driving-Point and Transfer Functions.

5. RC Driving Point Function Synthesis [23].

a. Foster Forms

An RC driving point impedance can be expressed in the form of a partial fraction expansion as follows:

$$Z(s) = k_\infty + \frac{k_0}{s} + \sum_i \frac{k_i}{s+\sigma_i} \tag{II.1}$$

Thus it can be synthesized as shown in Fig. II.5. This synthesis is known as first Foster form. Also for a given driving point admittance it is possible to expand Y(s)/s as in the form of Eq. II.1. Then Y(s) can be expressed as follows:

$$Y(s) = k_0 + k_\infty s + \sum_i \frac{k_i}{s+\sigma_i} \tag{II.2}$$

Thus it can be synthesized in second Foster form as shown in Fig. II.6.

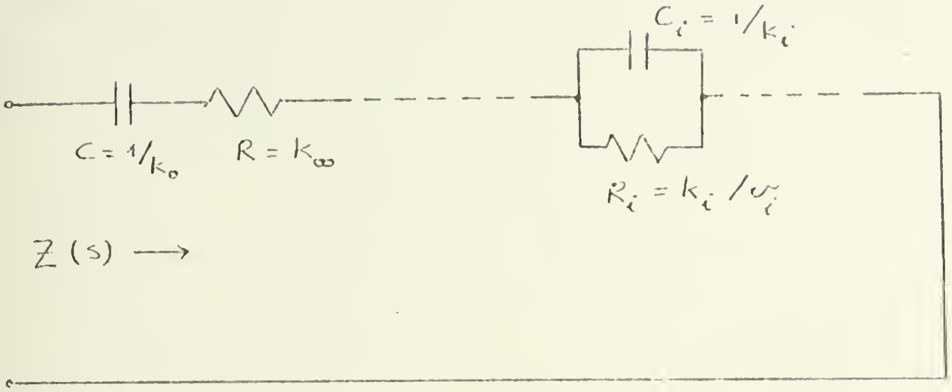


Figure II.5. First Foster Form Realization of RC Driving Point Impedances.

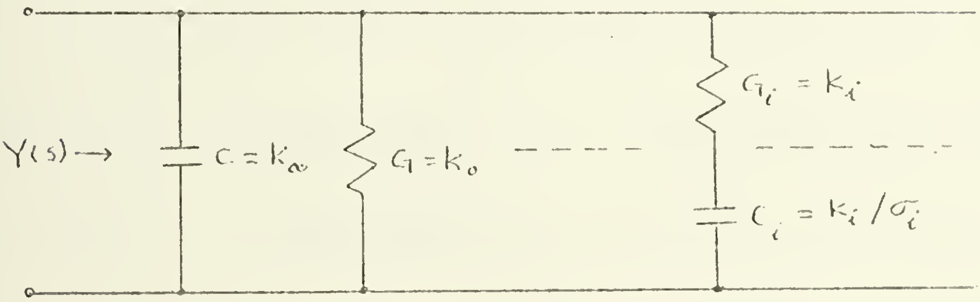


Figure II.6 Second Foster Form Realization of RC during Admittances

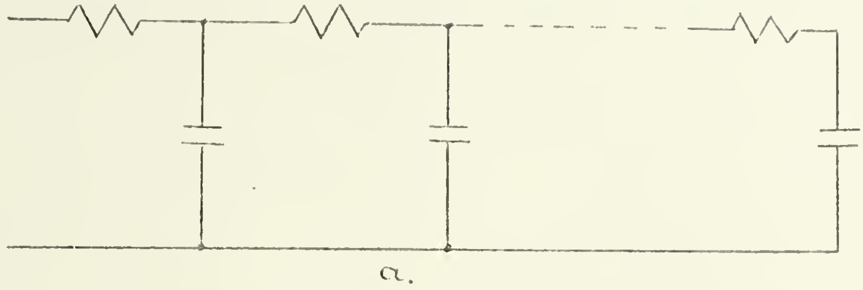
b. Cauer Forms

Another way of expanding the RC driving point functions is the continuous fraction expansion. This continuous fraction expansion can be synthesized in one of the two Cauer ladder networks shown in Fig. II.7.

6. Synthesis of Complex Transmission Zeros with Parallel RC Ladders.

Active RC synthesis frequently requires the realization of passive RC three-terminal networks. Two well known

methods are the Guillemin parallel ladder synthesis and the Fialkow-Gerst successive ladder development, which will subsequently be described.



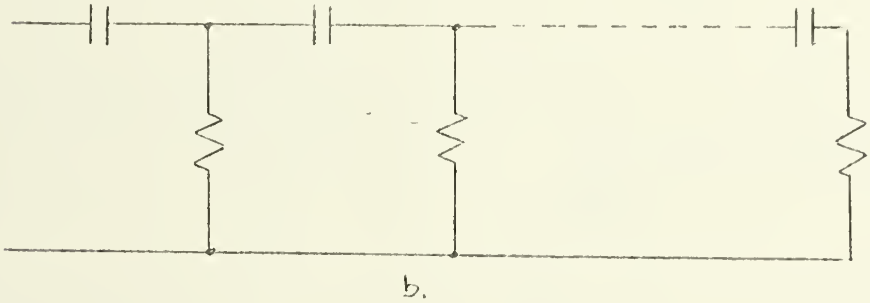


Figure II.7a - First Cauer Form of the RC Driving Point Functions.

" II.7b - Second Cauer Form of the RC Driving Point Functions.

Most of the active RC network synthesis techniques assume a predetermined structure together with its associated network function. The circuit function to be realized is compared with this function and network parameters are deduced from this identification thus completing the RC network synthesis. As a specific example consider the structure shown in Fig. II.8. This circuit consists of an operational amplifier with voltage current feedback provided by the passive RC three-terminal networks N_a and N_b, such that the voltage gain T(s) is given by

$$T(s) = -y_{12a}(s) / y_{12b}(s) \tag{II.3}$$

$y_{12a}(s)$ is the transfer admittance of the three terminal RC network N_a and $y_{12b}(s)$ is the transfer admittance of the

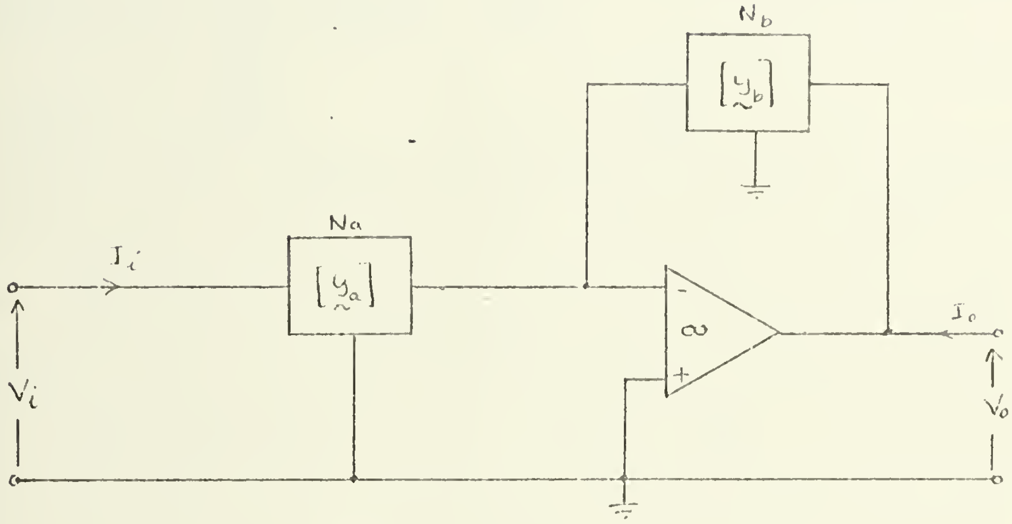


Figure II.8 An Infinite Gain Structure

other three terminal RC network N_b. When a rational polynomial $N(s)/Q(s)$ is given as the voltage gain $T(s)$ to be realized, then the ratio $N(s)/Q(s)$ must be identified with the term $-Y_{12a}(s)/Y_{12b}(s)$. The usual selection is $Y_{12a}(s) = N(s)/D(s)$ and $Y_{12b}(s) = Q(s)/D(s)$. When forming $T(s)$ only the numerator polynomials of the passive RC networks N_a and N_b namely $Y_{12a}(s)$ and $Y_{12b}(s)$ are retained hence the knowledge of $N(s)$ and $Q(s)$ is necessary but insufficient to synthesize either network N_a or N_b, since a

denominator polynomial D(s) is also required. The missing information can be found by a RC synthesis of one of the networks, say N_a in terms of $y_{12a}(s)$ and $y_{11a}(s)$.

As an example consider the numerator polynomial N(s) given for the synthesis of a three terminal RC network with parameter $y_{11}(s)$ and $y_{12}(s)$. Then

$$-y_{12}(s) = \frac{N(s)}{D(s)} \quad , \quad y_{11}(s) = \frac{P(s)}{D(s)}$$

where D(s) is the required denominator and P(s) is the numerator of $Y_{11}(s)$ which is arbitrary, but must be properly chosen. Note that no private poles are allowed in $y_{11}(s)$. If N(s) has all its roots on the negative real axis then the network can be synthesized by a single ladder. If N(s) has complex conjugate roots the network can be realized either by Guillemin's parallel ladder technique [24] or by Fialkow - Gerst's [25] successive ladder development. Now, these techniques are discussed as follows:

a. Guillemin's Technique

Given the numerator polynomial

$N(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots\cdots + a_1 s + a_0$ of the transfer admittance $-y_{12}(s)$, the sequence of Guillemin's technique is as follows:

(1) Divide the polynomial N(s) into smaller polynomials with real roots such that:

$N(s) = N_1(s) + N_2(s) + \cdots\cdots + N_N(s)$ where

$$N_1(s) = a_0 + a_1 s$$

$$N_2(s) = a_2 s^2 + a_3 s^3$$

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$$N_N(s) = a_{n-1} s^{n-1} + a_n s^n \qquad (\text{if } n \text{ is odd})$$

$$N_N(s) = a_n s^n \qquad (\text{if } n \text{ is even})$$

(2) Define a polynomial D(s) of degree n with negative real distinct roots such that

$$D(s) = (s+\sigma_1)(s+\sigma_2)\cdots\cdots(s+\sigma_n)$$

$$\sigma_i > 0 \qquad \sigma_1 < \sigma_2 < \cdots < \sigma_n$$

(3) Define N transfer admittances such that,

$$-y_{12}^{(1)}(s) = \frac{N_1(s)}{D(s)}, \quad -y_{12}^{(2)}(s) = \frac{N_2(s)}{D(s)}, \quad \cdots, \quad -y_{12}^{(N)}(s) = \frac{N_N(s)}{D(s)}$$

thus

$$-y_{12}(s) = -\sum_{i=1}^{N} y_{12}^{(i)}(s) = \frac{1}{D(s)} \sum_{i=1}^{N} N_i(s) = \frac{N(s)}{D(s)}$$

(4) Define a polynomial P(s) of degree n such that

$$P(s) = c_n s^n + c_{n-1} s^{n-1} + \cdots\cdots + c_1 s + c_0$$

$$= (s+\delta_1)(s+\delta_2)\cdots\cdots(s+\delta_n)$$

with the Fialkow - Gerst restrictions

$$c_i \geqslant a_i \qquad i = 0, 1, 2, \cdots, n$$

and the interlace requirement

$$\delta_1 < \sigma_1 < \delta_2 < \sigma_2 < \cdots\cdots < \delta_n < \sigma_n$$

The driving point parameter $y_{11}(s)$ is defined

$$y_{11}(s) = \frac{P(s)}{D(s)}$$

(5) Expand $y_{11}(s)$ into an RC ladder using the zero shifting technique [23] such that the zeros of the transfer admittances $-y_{12}^{(i)}(s)$ are realized in $y_{11}(s)$. This is done N times for each $y_{12}^{(i)}$ $i = 1,2,\cdots N$.

(6) This technique realizes each $-y_{12}^{(i)}(s)$ only to within a multiplicative constant K_i, such that

$$y_{12\,\text{Realized}}^{(i)}(s) = K_i\, y_{12\,\text{Desired}}^{(i)}(s)$$

Before connecting the N three terminal RC networks corresponding to each pair $y_{11}(s)$, $-y_{12}^{(i)}(s)$ in parallel their admittance levels must be adjusted because of the above mentioned constants K_i, which are found by letting $s = j\omega_i$ and considering the equality

$$y_{12\,\text{Realized}}^{(i)}(j\omega) = K_i\, y_{12\,\text{Desired}}^{(i)}(j\omega)$$

(7) Define a common scaling factor K such that

$$\frac{1}{K} = \sum_{i=1}^{N} \frac{1}{K_i} \qquad \text{(II.4)}$$

Then multiply the admittance level of the i^{th} network by

$\frac{K}{K_1}$ for $i = 1,2,\cdots, N$ and connect the networks in parallel.

The resulting network has a short circuit input admittance

of $y_{11}(s)$, and a transfer admittance of $Ky_{12}(s)$.

As an example [26] consider a given numerator polynomial N(s) of a transfer admittance $-y_{12}(s)$, namely

$$N(s) = s^2 + \sqrt{2}\, s + 1$$

N(s) has a pair of comples conjugate poles at $s = (-1 \pm j)/\sqrt{2}$

(1) N(s) is divided into two parts

$$N_1(s) = \sqrt{2}\, s + 1 , \quad N_2(s) = s^2$$

(2) D(s) is defined

$$D(s) = (s+1)(s+2)$$

(3) Two transfer admittance result, namely

$$-y_{12}^{(1)}(s) = \frac{\sqrt{2}\, s + 1}{(s+1)(s+2)} , \quad -y_{12}^{(2)}(s) = \frac{s^2}{(s+1)(s+2)} \qquad (II.5)$$

Thus

$$-y_{12}(s) = \frac{s^2 + \sqrt{2}\, s + 1}{(s+1)(s+2)}$$

(4) P(s) is defined

$$P(s) = \left(s + \frac{1}{\sqrt{2}}\right)\left(s + \sqrt{2}\right) = s^2 + \frac{3}{\sqrt{2}}\, s + 1$$

hence

$$y_{11}(s) = \frac{P(s)}{D(s)} = \frac{\left(s + \frac{1}{\sqrt{2}}\right)\left(s + \sqrt{2}\right)}{(s+1)(s+2)}$$

(5) $y_{11}(s)$ is expanded twice, first realizing the zeros $z_1 = -1/\sqrt{2}$, $z_2 = \infty$ of $-y_{12}^{(1)}(s)$ and secondly realizing the zeros $z_1 = z_2 = 0$ of $y_{12}^{(2)}(s)$.

These expansions are

1: $$y_{11}(s) = \cfrac{1}{\cfrac{0.76}{1 + \sqrt{2}\, s} + 1 + \cfrac{1}{2.94\, s + \cfrac{1}{0.24}}}$$

2: $$y_{11}(s) = \frac{1}{2} + \cfrac{1}{\cfrac{1}{0.31s} + \cfrac{1}{\cfrac{1}{2.23} + \cfrac{1}{18.9 + \cfrac{1}{0.0382s}}}}$$

and result in the structures shown in Fig. II.9 and II.10

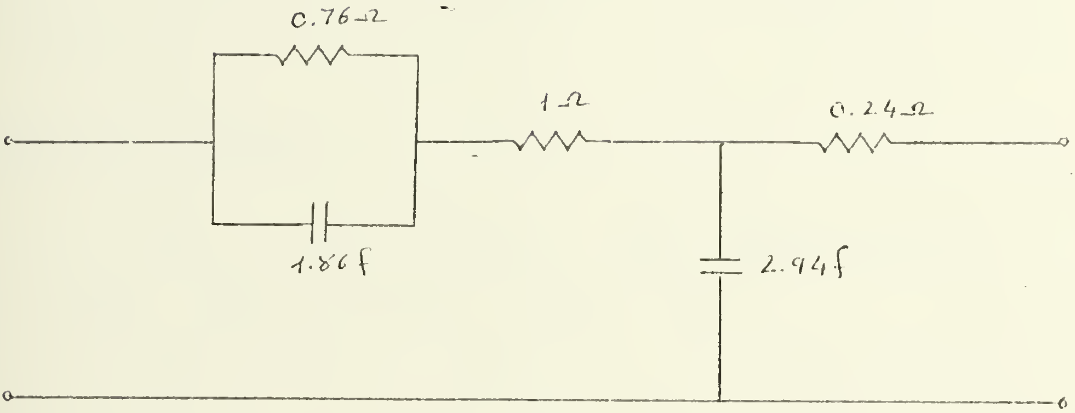


Figure II.9 First Expansion of $y_{11}(s)$

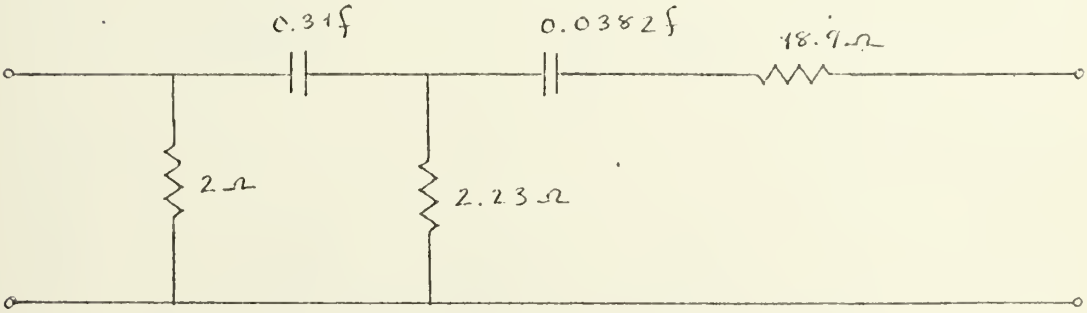


Figure II.10 Second Expansion of $y_{11}(s)$

(6) K_1 and K_2 are found as follows: The desired

values of $-y_{12}^{(i)}$ are given by Eq. II.5. The realized values are taken from Figs. II.9 and II.10,

1: $s = 0 \;, \; -y_{12}^{(1)}(0)_{\text{Realized}} = 0.5 \;, \; -y_{12}^{(1)}(0)_{\text{Desired}} = 0.5$.

Therefore $K_1 = 1$

2: $s = \infty \;, \; -y_{12}^{(2)}(\infty)_{\text{Realized}} = 0.053 \;, \; -y_{12}^{(2)}(\infty)_{\text{Desired}} = 1$.

Therefore $K_2 = 0.053$

(7) K, K/K_1, K/K_2 are found from Eq. II.4

$K = 0.0503 \quad K/K_1 = 0.0503 \quad K/K_2 = 0.95$

The admittance levels of the circuits are next adjusted by multiplying by K/K_1 and K/K_2. The final circuit is shown in Fig. II.11.

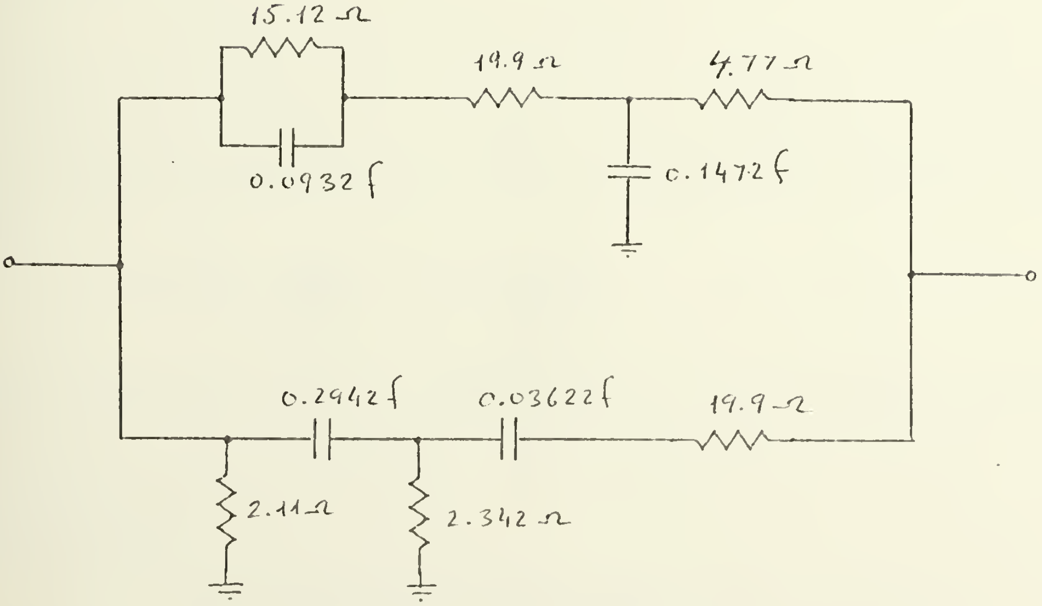


Figure II.11 Example of Guillemin's Technique

b. Fialkow - Gerst Technique

Guillemin's procedure imposes a gain factor K on the $y_{12}(s)$ which might not be desirable in some cases. The procedure which will be discussed next gives the exact required transfer admittance.

Given the numerator polynomial

$N(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots\cdots + a_1 s + a_0$ of the transfer admittance $-y_{12}(s)$ the sequence of the Fialkow - Gerst technique is as follows:

(1) Define a polynomial P(s) of degree n such that

$$P(s) = b_n s^n + b_{n-1} s^{n-1} + \cdots\cdots + b_1 s + b_0$$

$$= (s+\delta_1)(s+\delta_2)\cdots\cdots(s+\delta_n)$$

where $\quad b_i \geqslant a_i \quad , \delta_i > 0$ (II.6)

and

$$\delta_1 < \delta_2 < \cdots\cdots < \delta_n$$

as mentioned before in the Guillemin procedure. Then define the voltage transfer of a three terminal passive RC network as

$$T(s) = \frac{-N(s)}{P(s)} = -\frac{y_{12}(s)}{y_{22}(s)}$$

The polynomial P(s) is identified as the numerator polynomial of $y_{22}(s)$. Note that Eq. II.6 satisfies the Fialkow - conditions.

(2) Define a polynomial D(s) of degree n-1 such that

$$D(s) = (s+\sigma_1)(s+\sigma_2)\cdots\cdots(s+\sigma_{n-1})$$

where

$$\delta_1 < \sigma_1 < \delta_2 < \sigma_2 < \cdots\cdots < \sigma_{n-1} < \delta_n$$

Then define the driving point admittance $Y_{22}(s)$ as

$$y_{22}(s) = \frac{P(s)}{D(s)}$$

(3) Expand $Y_{22}(s)$ in the canonical form

$$y_{22}(s) = s + c + \sum_{i=1}^{n-1} \frac{c_i s}{s+\sigma_i}$$

Then split $Y_{22}(s)$ into two parts

$$y_{22}(s) = \left(s + \sum_{i=1}^{n-1} \frac{\alpha_i c_i s}{s+\sigma_i} \right) + \left(c + \sum_{i=1}^{n-1} \frac{(1-\alpha_i) c_i s}{s+\sigma_i} \right) \qquad \text{(II.7)}$$

where $0 < \sigma_i < 1$. Define two driving point admittances $Y_{22}^{(1)}(s)$ and $Y_{22}^{(2)}(s)$

$$y_{22}^{(1)}(s) = \left(s + \sum_{i=1}^{n-1} \frac{\alpha_i c_i s}{s+\sigma_i} \right) \qquad \text{(II.8a)}$$

$$y_{22}^{(2)}(s) = \left(c + \sum_{i=1}^{n-1} \frac{(1-\alpha_i) c_i s}{s+\sigma_i} \right) \qquad \text{(II.8b)}$$

The parallel combination of three terminal passive RC networks shown in Fig. II.12 implies Eqs. II.7 and II.8.

(4) Multiply $Y_{22}^{(1)}(s)$ and $Y_{22}^{(2)}(s)$ by D(s). Note that this will give P(s) in two parts

$$P(s) = y_{22}(s) D(s) = y_{22}^{(1)}(s) D(s) + y_{22}^{(2)}(s) D(s)$$

More specifically

$$P(s) = P_1(s) + P_2(s) = \left(s \sum_{i=0}^{n-1} b'_{n-i} s^{n-i-1} \right) + \left(\sum_{i=1}^{n} b''_{n-i} s^{n-i} \right)$$

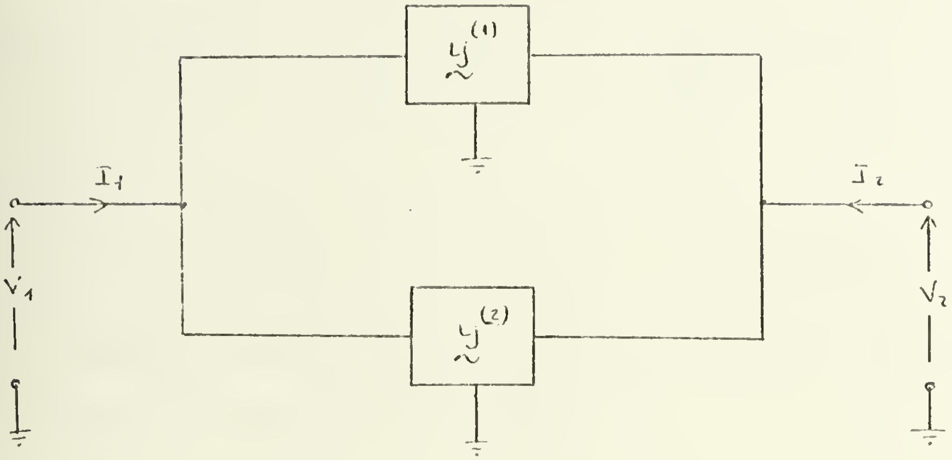


Figure II.12 The Fialkow - Gerst 3-Terminal Network

(5) Split the numerator polynomial N(s) into two parts

$$N(s) = N_1(s) + N_2(s) = \left(s \sum_{i=0}^{n-1} a'_{n-i} s^{n-i-1} \right) + \left(\sum_{i=1}^{n} a''_{n-i} s^{n-i} \right)$$

where $0 \leq a'_i \leq b'_i$, $0 \leq a''_i \leq b''_i$ (II.9)

Note that $a'_i + a''_i = a_i$ and $b'_i + b''_i = b_i$. Since $a_i \leq b_i$ the Eq. II.9 can always be satisfied.

(6) Identify the following terms

$$y_{22}^{(1)}(s) = \frac{P_1(s)}{D(s)} \quad , \quad y_{22}^{(2)}(s) = \frac{P_2(s)}{D(s)}$$

$$y_{12}^{(1)}(s) = -\frac{N_1(s)}{D(s)} \quad , \quad y_{12}^{(2)}(s) = -\frac{N_2(s)}{D(s)}$$

$$T_1(s) = -\frac{y_{12}^{(1)}(s)}{y_{22}^{(1)}(s)} = -\frac{N_1(s)}{P_1(s)}, \quad T_2(s) = -\frac{y_{12}^{(2)}(s)}{y_{22}^{(2)}(s)} = -\frac{N_2(s)}{P_2(s)} \tag{II.10}$$

$$T(s) = -\frac{N(s)}{P(s)} = -\frac{N_1(s)+N_2(s)}{P_1(s)+P_2(s)} = -\frac{y_{12}^{(1)}(s)+y_{12}^{(2)}(s)}{y_{22}^{(1)}(s)+y_{22}^{(2)}(s)} \tag{II.11}$$

Note that the Eq. II.10 gives the voltage transfer functions of the sub-networks shown in Fig. II.12, and Eq. II.11 gives the voltage transfer function of the overall network. Eq. II.10 can be rewritten as follows

$$T_1(s) = -\frac{s\sum_{i=0}^{n-1} a'_{n-i}\, s^{n-i-1}}{s\sum_{i=0}^{n-1} b'_{n-i}\, s^{n-i-1}} = -\frac{a'_n s^{n-1} + \cdots + a'_1}{b'_n s^{n-1} + \cdots + b'_1}$$

$$T_2(s) = -\frac{\sum_{i=1}^{n} a''_{n-i}\, s^{n-i}}{\sum_{i=1}^{n} b''_{n-i}\, s^{n-i}} = -\frac{a''_{n-1} s^{n-1} + \cdots + a''_0}{b''_{n-1} s^{n-1} + \cdots + b''_0}$$

Note that both $T_1(s)$ and $T_2(s)$ are at least one degree simpler than $T(s)$.

(7) Reduce the $Y_{22}^{(1)}(s)$ and $Y_{33}^{(2)}(s)$ to the following form

$$\frac{1}{y_{22}^{(1)}(s)} = \frac{D(s)}{s\sum_{i=0}^{n-1} b'_{n-i}\, s^{n-i-1}} = \frac{1}{Cs} + \frac{1}{Y_{22}^{(1)}(s)}$$

$$\frac{1}{y_{22}^{(2)}(s)} = \frac{D(s)}{\sum_{i=1}^{n} b''_{n-i}\, s^{n-i}} = R + \frac{1}{Y_{22}^{(2)}(s)}$$

This last reduction is shown in Fig. II.13.

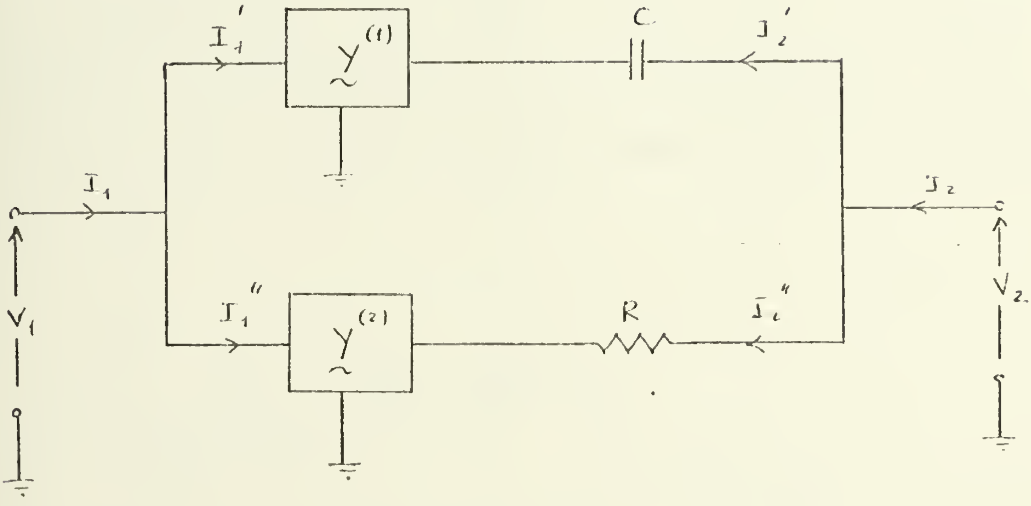


Figure II.13 Development of Fialkow - Gerst Synthesis

Note that the two new subnetworks $Y^{(1)}$ and $Y^{(2)}$ have the same transfer functions $T_1(s)$ and $T_2(s)$ since

$$T_1(s) = \left.\frac{V_o}{V_1}\right|_{I_2'=0} \quad \text{and} \quad T_2(s) = \left.\frac{V_o}{V_1}\right|_{I_2''=0}$$

(8) Find the new transfer admittances as follows:

$$-Y_{12}^{(1)}(s) = T_1(s) \cdot Y_{22}^{(1)}(s)$$

$$-Y_{12}^{(2)}(s) = T_2(s) \cdot Y_{22}^{(2)}(s)$$

At this point of synthesis one cycle is complete. The transfer functions and the admittance matrix of the new sub-networks are known. If the transfer functions and admittances are of first degree they are realized as ladders. If their degree is greater than one, then the same cycle is repeated from beginning of the Step (3) for the sub-networks. Thus, the result is an expanding parallel ladder structure as shown in Fig. II.14.

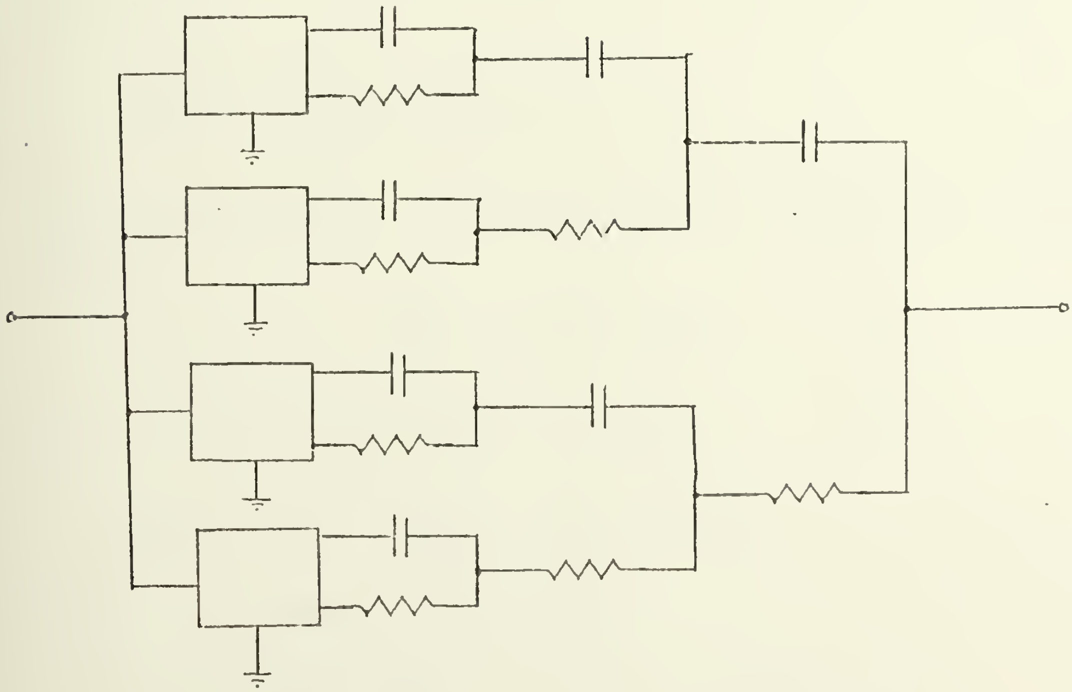


Figure II.14 Fialkow - Gerst Three Terminals RC Network.

As an example consider a given numerator polynomial of a transfer admittance $-y_{12}(s)$, namely

$$N(s) = s^2 + \sqrt{2}\, s + 1$$

A polynomial P(s) is selected

$$P(s) = (s+1)(s+3)$$

Hence

$$T(s) = -\frac{s^2 + \sqrt{2}\, s + 1}{s^2 + 4s + 3}$$

$y_{22}(s)$ is defined by selecting a polynomial

$$D(s) = s + 2$$

Thus

$$y_{22}(s) = \frac{s^2 + 4s + 3}{s + 2} = s + \frac{3}{2} + \frac{\frac{1}{2} s}{s+2}$$

$y_{22}(s)$ is decomposed into two parts

$$y_{22}^{(1)}(s) = \left(s + \frac{\frac{1}{4} s}{s+2} \right)$$

$$y_{22}^{(2)}(s) = \left(\frac{3}{2} + \frac{\frac{1}{4} s}{s+2} \right)$$

Multiplying $y_{22}^{(1)}(s)$ and $y_{22}^{(2)}(s)$ by D(s) gives the decomposition of P(s)

$$P(s) = s\left(s + \frac{9}{4} \right) + \left(\frac{7}{4} s + 3 \right)$$

N(s) is also decomposed arbitrarily

$$N(s) = s\left(s + \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} s + 1 \right)$$

Hence $T_1(s)$ and $T_2(s)$ are found

$$T_1(s) = \frac{(s + 1/\sqrt{2})}{(s + 9/4)} \quad , \quad T_2(s) = \frac{(1/\sqrt{2}\, s + 1)}{(7/4\, s + 3)}$$

$y_{22}^{(1)}(s)$ and $y_{22}^{(2)}(s)$ are reduced and $y_{22}^{(1)}(s)$ and $y_{22}^{(2)}(s)$ are determined according to

$$\frac{1}{y_{22}^{(1)}(s)} = \frac{2+s}{\frac{9}{4}s+s^2} = \frac{1}{1.125\,s} + \frac{1}{Y_{22}^{(1)}(s)}$$

$$\frac{1}{y_{22}^{(2)}(s)} = \frac{s+2}{\frac{7}{4}s+3} = 0.572 + \frac{1}{Y_{22}^{(2)}(s)}$$

where

$$Y_{22}^{(1)}(s) = 9s + 20.25 \;, \quad Y_{22}^{(2)}(s) = 6.12s + 10.5$$

and

$$Y_{12}^{(1)}(s) = T_1(s)\,Y_{22}^{(1)}(s) = 9s + 6.35 \,, \quad Y_{12}^{(2)}(s) = T_2(s)\,Y_{22}^{(2)}(s) = 2.48s + 3.5$$

Since the new transfer functions and admittances are of first degree there is no need to recycle again. Thus the final network is given in Fig. II.15.

Both Guillemin's and Fialkow - Gerst's techniques are long and tedious. The Fialkow - Gerst synthesis becomes very cumbersome for degrees greater than four. Guillemin's technique is done in one cycle, however if the degree is greater than three it is only possible to find K_1 and K_N by substituting a value of $s=0$ and $s=\infty$. For K_i, $i=2,3,\cdots,N-1$ a numerical value for $s=j\omega_i$ must be substituted and this is also a long and tedious process. Further, the zeros of $N(s)$ which lie in the right half of the s-plane cannot be realized by these techniques. For the second degree functions it is

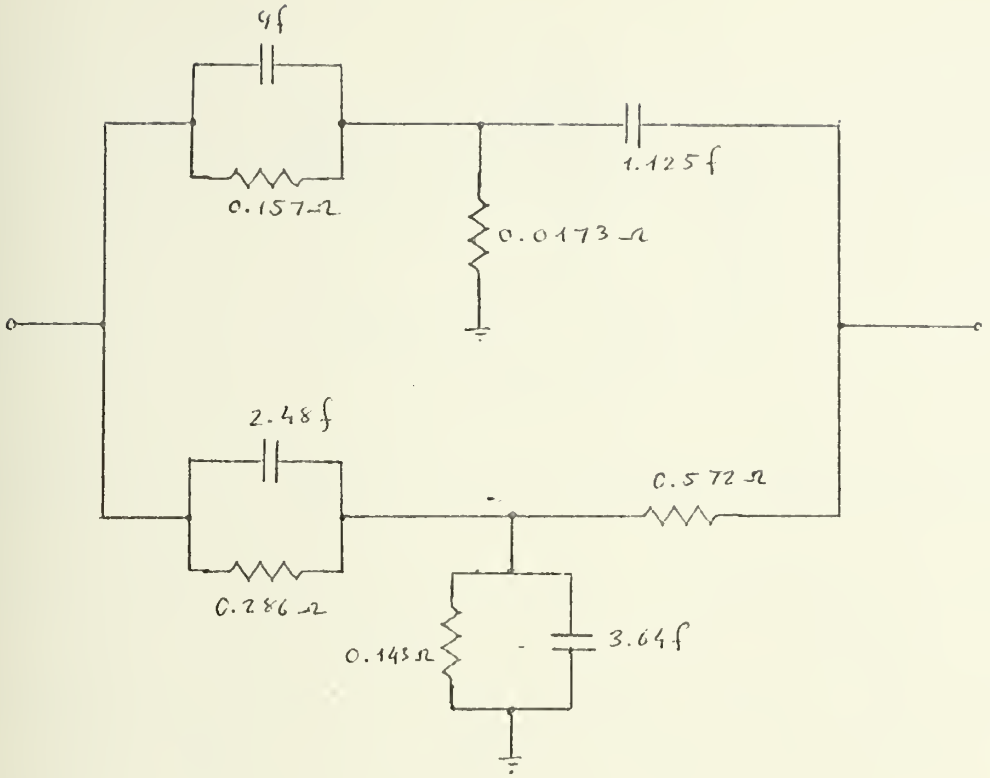


Figure II.15 The Example of Fialkow - Gerst Synthesis

possible to use some well investigated circuits such as the twin-T and bridged-T circuit. Such circuits and their characteristics can be found in literature [14, 27, 28].

B. STABILITY

The necessary and sufficient conditions for stability were mentioned in the introduction. Consider now a single loop negative feedback circuit shown in Fig. II.16, so as to study the stability of active RC structures.

The closed-loop gain of this system is:

$$\frac{V_o}{V_i} = \frac{A_o(s)}{1 + A_o(s)\beta}$$

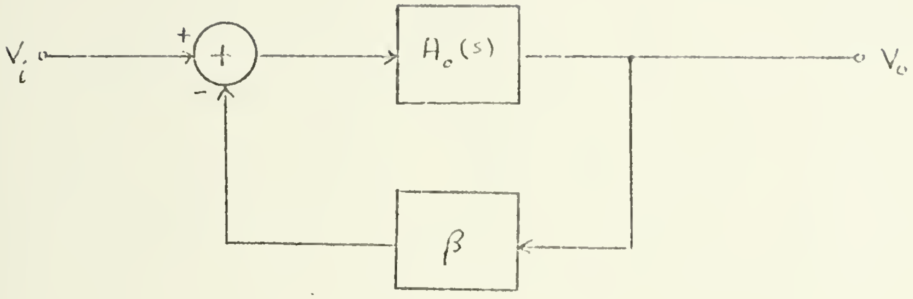


Figure II.16 A Single Loop Negative Feedback System

where $A_o(s)$ is the open-loop gain and $A_o(s)\cdot\beta$ is termed the loop gain of the system. If $[1+A_o(s)\cdot\beta]$ vanishes at some frequency ω_o, then $\frac{V_o}{V_i}$ will become infinite and there is a $j\omega$ axis pole at $\omega=\omega_o$, which indicates instability of the closed loop system. If $[1+A_o(j\omega_o)\beta] = 0$, then $|A_o(j\omega_o)\cdot\beta| = 1$. And so $\angle A_o(j\omega_o)\cdot\beta = \pi$ or an odd multiple of π. The Bode criterion for strict stability follows from these remarks:

A single-loop feedback circuit is strictly stable if at the unity crossover frequency ω_o of the loop gain, the phase angle is less than 180°.

Assume that the Bode plot of the loop gain is as shown in Fig. II.17. The figure shows that the phase of the loop gain reaches 180° between ω_2 and ω_3 while the gain rolls off 12db/octave. Therefore, Bode's criterion can be restated as follows: A single loop feedback circuit is strictly stable, if at the zero db crossover frequency ω_o of the loop gain, the loop gain has a slope of less than 12db/octave.

If there are two active RC structures which realize a circuit function and if one of them is unstable, then certainly

the stable one must be selected unless it is desired to build an oscillator. However, if both of the structures are stable, then the more stable one must be selected. The measure of stability called the gain margin and phase margin is also shown in Fig. II.17. The gain margin is defined as the gain increase needed to drive the system into instability. The phase margin is defined as the phase difference between zero crossover frequency and 180°. In most cases a satisfactory phase margin guarantees a good gain margin. However, cases exist in which both margins should be considered.

C. SENSITIVITY

The sensitivity function which was first mentioned in the introduction is now defined.

1. Sensitivity Function

The sensitivity $S_k^{T(s)}$ of the network function $T(s,k)$ due to the variation of the parameter k is [14]

$$S_k^{T(s)} \triangleq \frac{dT(s,k)/T(s,k)}{dk/k} \qquad \text{(II.12)}$$

The sensitivity function assumes an incremental change in parameter k. From Eq. II.12 two relations can be derived:

$$S_{1/k}^{T} = -S_k^{T}$$

Further if $k_2 = k_2(k_1)$ then

$$S_{k_1}^{T} = S_{k_2}^{T} \cdot S_{k_1}^{k_2}$$

2. Gain Sensitivity and Phase Sensitivity

For the sinusoidal steady state $s = j\omega$ and so $T(s) = T(j\omega)$. Hence

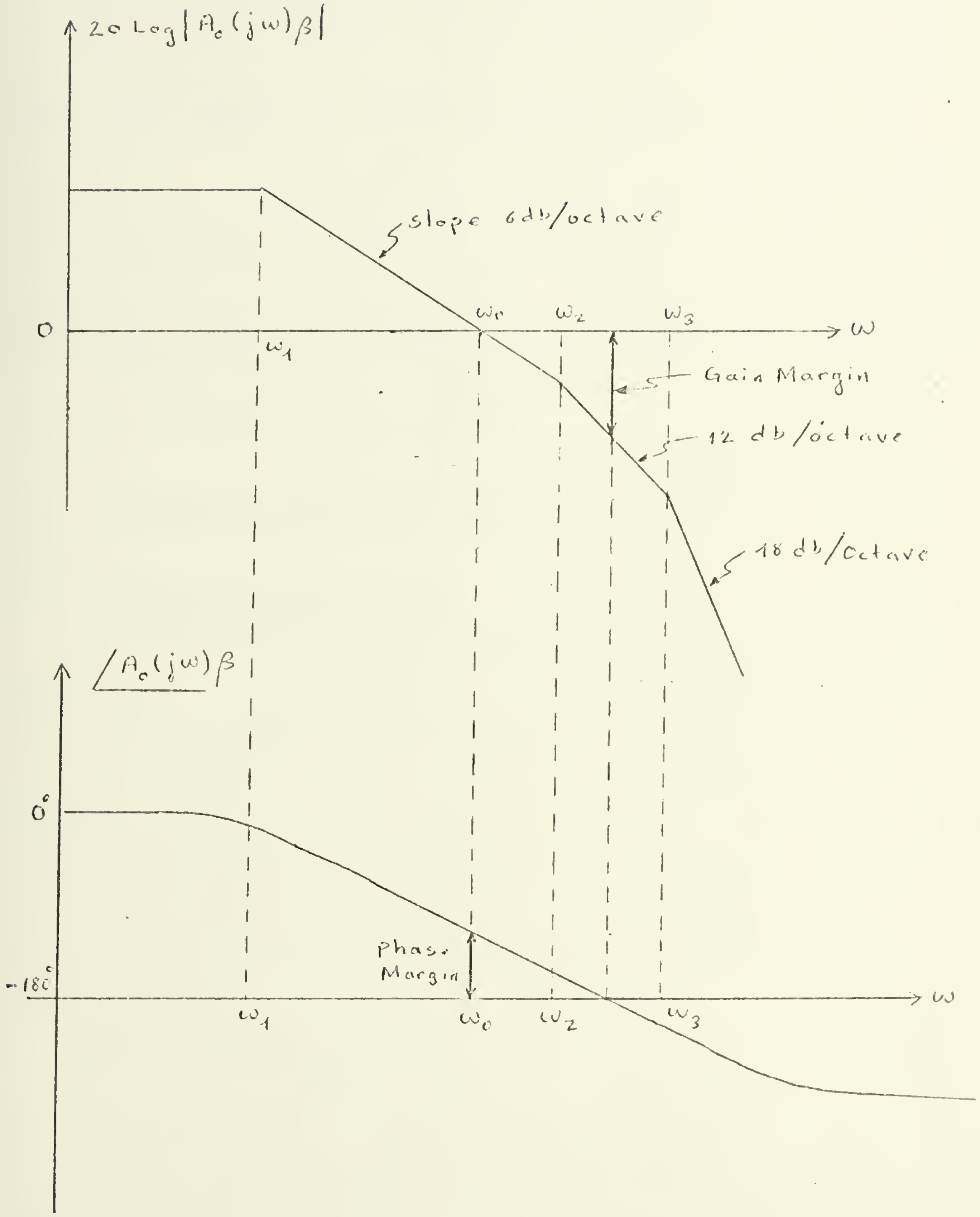


Figure II.17 The Bode Plot of the Loop Gain of the Feedback System Shown in Fig. II.16

$$S_k^{T(j\omega)} \triangleq \frac{dT(j\omega)/T(j\omega)}{dk/k}$$

writing T(jω) which is complex, in polar form yields

$$T(j\omega) = |T(j\omega)| e^{j\arg[T(j\omega)]}$$

The gain and phase sensitivities are now defined as follows:

Gain sensitivity $= S_k^{|T(j\omega)|}$

Phase sensitivity $= S_k^{\exp[j\arg T(j\omega)]}$

3. Polynomial Sensitivity

For some applications only numerator or denominator polynomials of a circuit function may be of interest. For example, in bandpass transfer functions the denominator polynomial of the transfer function determines the response. For this case the polynomial sensitivity is defined as follows: The polynomial sensitivity of a polynomial D(s,k) with respect to the variable parameter k is:

$$S_k^{D(s,k)} \triangleq \frac{dD(s,k)/D(s,k)}{dk/k}$$

4. Zero Sensitivity and Pole Sensitivity

It has previously been mentioned that the motion of poles and zeros from the left half s-plane to the right half s-plane can render a circuit unstable. Hence zero and pole sensitivity is of great importance in active RC network

theory. For example in a highly selective bandpass filter the change of location of the poles can be very crucial in two respects. First the change of pole location causes the resonance frequency to drift which might violate the specification to be met. Second, and more important, if the selectivity is high then the original locations of the poles are near to the jω axis and a drift may cause them to move into the right half of the s-plane. In that case, the system becomes unstable. Hence it is important to minimize pole (zero) sensitivity.

Pole (zero) sensitivity is defined as follows [14]: Let $s = p_i$ be a pole (zero) of T(s,k) when k takes its nominal value. The pole (zero) sensitivity of T(s,k) is then defined as:

$$S_k^{p_i} \triangleq \left. \frac{dp_i}{dk/k} \right|_{s=p_i}$$

5. Coefficient Sensitivity

The coefficients of the numerator and the denominator polynomial of a network are implicit functions of the variable parameter k. The sensitivity of the coefficients with respect to the variation of parameter k is defined as follows:

Let

$$P(s,k) = \sum_{j=0}^{n} a_j s^j$$

be a polynomial of interest. The coefficient sensitivity is then defined as:

$$S_k^{a_j} \triangleq \frac{da_j/a_j}{dk/k}$$

6. Q Sensitivity

The Q sensitivity with respect to a variable network parameter k is defined as

$$S_k^Q \triangleq \frac{dQ/Q}{dk/k}$$

The Q sensitivity and the pole sensitivity of a complex conjugate pair of poles are related [14]. Since

$$Q = \frac{\omega_i}{2\sigma_i}$$

it can be proven that [29]

$$S_k^Q = -2Q\, Im\left[\frac{S_k^{P_i}}{P_i}\right]$$

7. Interrelations between Sensitivities

Six kinds of sensitivities have been defined above. They are not completely independent from each other as will be shown now.

a. Pole and zero sensitivities. These are related to the sensitivity function of a network function. Consider a network function T(s).

$$T(s) = K \frac{\prod_{i=1}^{m} (s - z_i)}{\prod_{j=1}^{n} (s - p_j)}$$

where k is the gain factor or multiplier. By simple substitutions it can be proven that the relation between the pole, zero sensitivities and sensitivity function is as follows [30].

$$S_k^{T(s)} = S_k^{K} - \sum_{i=1}^{m} \frac{S_k^{z_i}}{(s-z_i)} + \sum_{j=1}^{n} \frac{S_k^{p_j}}{(s-p_j)}$$

b. Polynomial root sensitivity:

Sum of the root sensitivities of a polynomial will be presented with the aid of bilinear forms of a network function.

Let k be a variable network parameter and T(s,k) any network function of the form:

$$T(s,k) = \frac{N(s,k)}{D(s,k)}$$

It is possible to express (T(s,k) as follows:

$$T(s,k) = \frac{N(s,k)}{D(s,k)} = \frac{N_1(s) + kN_2(s)}{D_1(s) + kD_2(s)}$$

and this is called a bilinear form. Consider a polynomial P(s).

$$P(s,k) = \sum_{j=0}^{n} a_j s^j = \prod_{j=1}^{n} (s - p_j)$$

$$= (b_n + kc_n)s^n + (b_{n-1} + kc_{n-1})s^{n-1} + \cdots$$

$$\cdots + (b_1 + kc_1)s + (b_0 + kc_0)$$

The sum of the root sensitivities of P(s,k) with respect to the variable parameter k can be expressed as follows [31].

$$\sum_{j=1}^{n} S_k^{p_j} = \frac{k(c_n b_{n-1} - c_{n-1} b_n)}{(b_n + k c_n)^2}$$

The relations between the root sensitivities and the coefficient sensitivities follows:

[14] $$\sum_{j=1}^{n} S_k^{p_j} = \left(S_k^{a_n} - S_k^{a_{n-1}} \right) \cdot \frac{a_{n-1}}{a_n}$$

[14] $$\sum_{j=1}^{n} \frac{S_k^{p_j}}{p_j} = (-1)^n \cdot \left(S_k^{a_0} - S_k^{a_n} \right)$$

[32] $$S_k^{p_j} = -L_1^{(j)} \left[\sum_{i=0}^{n} a_i S_k^{a_i} (p_j)^i \right]$$

where

$$L_1^{(j)} = \left. \frac{1}{dP(s)/ds} \right|_{s=p_j}$$

D. COMPARISON OF DIFFERENT RC ACTIVE STRUCTURES

The active devices which are used in RC active network synthesis have already been presented in the introduction. Each device has its own advantages and disadvantages. For example, operational amplifiers are cheap and readily available, on the other hand, they are not ideal because their

gain is finite and neither the input nor the output impedance are infinite or zero respectively. Moreover extreme care must be taken to ensure circuit stability. Nevertheless successful designs do exist and it appears that the RC - operational amplifier combination will become the standard filter building block. Moschytz's [11] and Newcomb's [12] designs may be mentioned in this respect.

Controlled sources have also proven successful as active devices. Some of the circuits listed by Sallen and Key [33] have excellent sensitivities. The finite gain amplifiers necessary for these structures can be built with infinite gain operational amplifiers connected in feedback configuration. The overall finite gain is then the ratio of two resistances and this procedure leads to excellent gain stability.

Gyrators are also good devices to build active RC structures in the low-frequency range, provided that they have small input and output losses. In the introduction it was found that they can provide high Q's with careful design.

NIC's have an inherent disadvantage because in good designs they suffer from sensitivities which are proportional to Q. This may be seen by considering a second order denominator polynomial with high Q.

$$D(s) = s^2 + 2\sigma s + \omega_n^2$$

This function can be decomposed into two parts, one of which has negative real roots and the other has the root s = 0.

$$D(s) = s^2 + 2\sigma s + \omega_n^2 = (s^2 + \partial' s + \omega_n^2) - k\eta s$$

where $2\sigma = \gamma - k\eta$ and k is the conversion factor of NIC. Such a decomposition has the best pole sensitivity as proved by Horowitz [34].

Now the Q can be expressed as

$$Q \simeq \frac{\omega_n}{2\sigma} = \frac{\omega_n}{\gamma' - k\eta}$$

hence

$$S_k^Q = \frac{\gamma'}{2\sigma} - 1 .$$

Because the roots of $(s^2 + \gamma' s + \omega_n^2)$ are negative and real $\gamma' > 2\omega_n$ and so $S_k^Q > \frac{2\omega_n}{2\sigma} - 1$

or

$$S_k^Q > 2Q - 1 \tag{II.12}$$

Equation II.12 gives the best Q sensitivity which can be obtained. Therefore the RC-NIC circuits are not suitable for high Q realization.

Another important factor concerns the sensitivities with respect to passive element variations.

If the circuit is to be integrated this becomes very important because the tolerances of diffused integrated circuit resistors vary between 10% - 20%. Also the temperature coefficients of integrated resistors are of the order of 1000 - 3000 ppm/C°. However if the circuit function of an active RC network can be made to depend on resistance ratios instead of absolute resistance values, then it is possible to achieve 1% ratio tolerances in integrated realizations [35]. Furthermore resistors will track each other with temperature. The passive element values must also be carefully chosen so as to be suitable for integration.

Reasonable resistor values lie between 100Ω to 30 KΩ [35]. Capacitor values lie typically between 500 pF to 5000 pF [36] in monolithic semiconductor design, but lower values of the order of 10 to 100 pF are possible with metal-semiconductor technology.

E. IC OPERATIONAL AMPLIFIER

Since the operational amplifier is the basic active building block of all circuits described in this thesis it is necessary to discuss practical operational amplifiers in more detail than was done in the introduction. For example, in a practical operational amplifier one of the significant characteristics is input offset voltage. In a practical amplifier, a finite output voltage appears even though the input terminals are shorted together. This is undesirable as it degrades common mode rejection. In practice it leads to limiting of the output signal swing. The DC input voltage that causes the output to go to zero is called the input offset voltage. Its value is usually given in data sheets for room temperatures. For other environments, it has to be determined experimentally.

The amplifier's finite input impedance is another source of error. Each input requires a small bias current and this current is not the same for both inputs. The difference is expressed as input offset current, and it is specified relative to the average input bias current. Another important deviation from the ideal is the change in voltage, current and gain levels with temperature, and this is called drift.

Further, input impedance is defined separately for differential and for common mode operation. Differential input resistance is defined as the resistance into either input terminal with the other terminal grounded. Common mode resistance is the resistance between inputs and ground with both inputs shorted together. Common mode input resistance is usually high (20MΩ to 100MΩ [37]) and can be assumed infinite in most cases. However, the differential input resistance may be low as a few thousand ohms, and does affect the closed loop gain. Output resistance is the resistance seen looking into the output terminal with the output voltage signal at zero. Its value is usually about one hundred ohms.

The open loop voltage gain is the ratio of the maximum output swing to the change in input voltage needed to drive the output from zero to this maximum. Ideally the gain from one input to the output should be equal and opposite to the gain from the other. If the same voltage were applied to both inputs, the output should remain zero. In practice equal changes in both inputs produce an output. Common mode rejection ratio is the ratio of differential mode gain to common mode gain.

Maximum output power usually occurs at some output voltage below the rated maximum. Maximum output power is the product of the voltage at this point and the maximum output current. The maximum voltage that can be applied at either input of the amplifier with respect to ground is the input voltage range. Exceeding the specified input voltage range can damage the amplifier.

The most important characteristic which deviates from the ideal one is the open loop frequency response, which is now discussed.

1. Open Loop Gain

The open loop gain of a practical operational amplifier is a function of frequency. It can be approximated by the expression

$$A_o(s) = \frac{A_o}{(s+j\omega_1)(s+j\omega_2)(s+j\omega_3)} \qquad \text{(II.13)}$$

where ω_1, ω_2, and ω_3 are called the break points of the characteristic. The Bode plot of Eqn. II.13 is shown in Fig. II.18 with solid lines.

The closed loop gain of a single loop negative feedback system was given in the section on stability as

$$A(s) = \frac{A_o(s)}{1 + A_o(s)\beta}$$

The loop gain was defined as $A_o(\omega)\cdot\beta$ and may be expressed in decibels

$$20\,\mathrm{Log}\left[A_o(s)\cdot\beta\right] = 20\,\mathrm{Log}\left[A_o(s)\right] - 20\,\mathrm{Log}\left(1/\beta\right)$$

and the magnitude of this loop gain is shown on Fig. II.18. As may be seen the loop gain has a crossover frequency $\omega_{o(\text{uncomp.})}$ with the slope of 12 db/oct., therefore the feedback system is unstable by Bode's criterion. If the amplifier's open loop gain is compensated as shown with dashed lines in Fig. II.18, then the rate of closure, i.e., the difference of slopes of 20 $\log(\frac{1}{\beta})$ and 20 log $A_o(s)$, or the slope of the loop gain at the new crossover frequency $\omega_{o(\text{comp.})}$ would

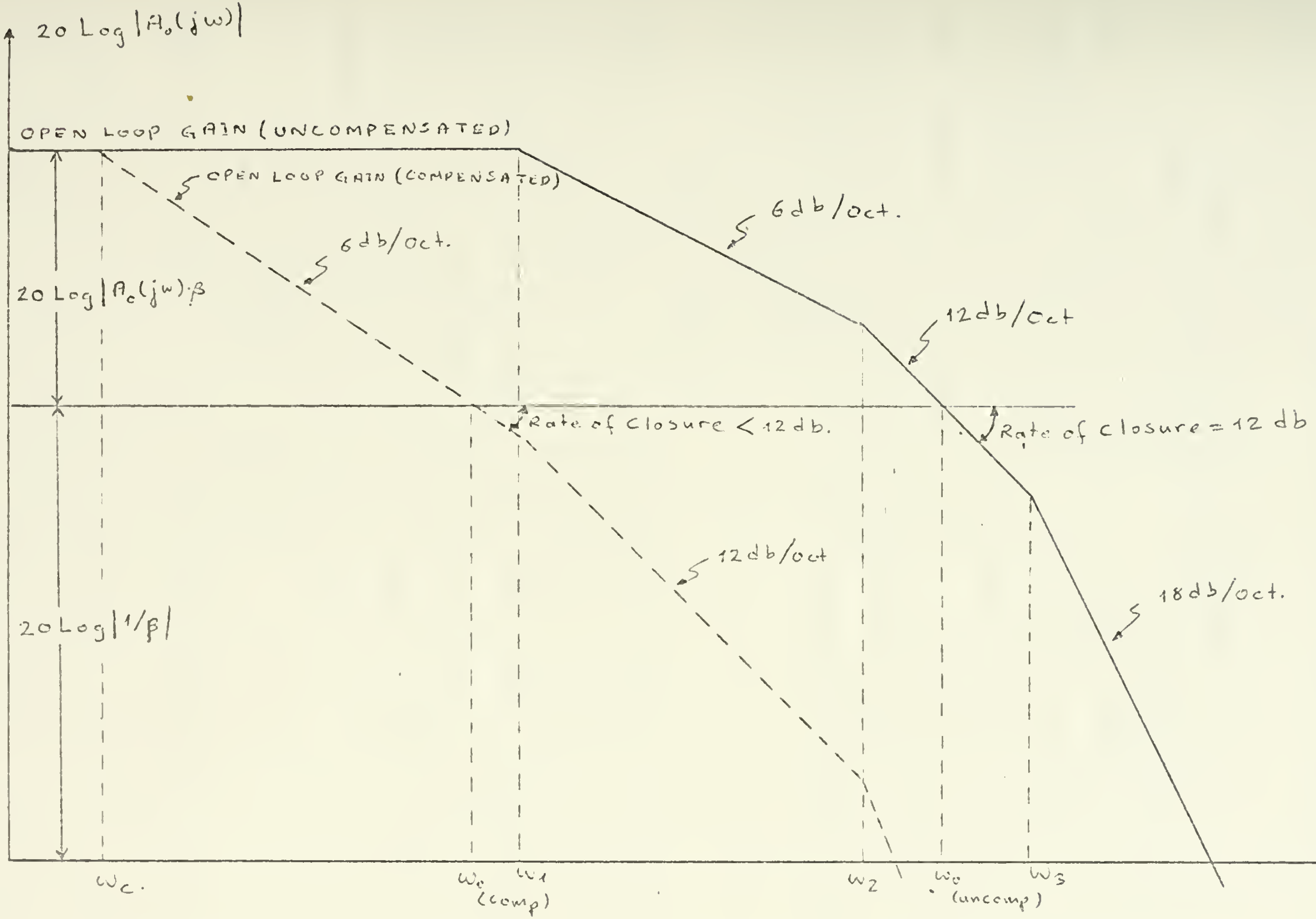


Figure II.18 Frequency Response of an Operational Amplifier and the Effect of Compensation.

become less than 12 db/oct. and the system becomes stable. As a conclusion then the open loop gain of an operational amplifier must be compensated before one attempts to design an active RC network.

2. Compensation of Open Loop Gain

There are two methods of compensating the open loop gain of an operational amplifier namely internal and external compensation. The practical operational amplifier in general is provided with at least one terminal where additional network elements can be connected for internal compensation. Consider such an operational amplifier shown in Fig. II.19a. If an external capacitor C_1 is connected as shown then the open loop gain becomes

$$A_{Comp}(s) = \frac{A_0(s)\,\omega_c}{s+\omega_c}$$

where $\omega_c = 1/R_1C_1$. By adjusting the value of the capacitor C_1 it is possible to choose the break point ω_c such that at ω_1 the loop gain became less than unity. This compensation gives the dashed curve shown in Fig. II.18. Another internal compensation scheme is shown in Fig. II.19b. This compensation gives an open loop gain

$$A_{Comp}(s) = \frac{R_2\,A_0(s)}{R_1+R_2}\cdot\frac{(s+\omega_{c2})}{(s+\omega_{c1})}$$

where $\omega_{c2} = 1/R_2C_1$, $\omega_{c1} = 1/(R_1+R_2)C_1$ and so $\omega_{c2} \gg \omega_{c1}$.

If ω_{c2} is chosen to be equal to ω_1 the compensated open loop gain will have a slope of 6 db/oct. from ω_{c1} to ω_2. A proper selection of ω_{c1} will assure that the 0 db crossover frequency occurs at 6 db/octave slope.

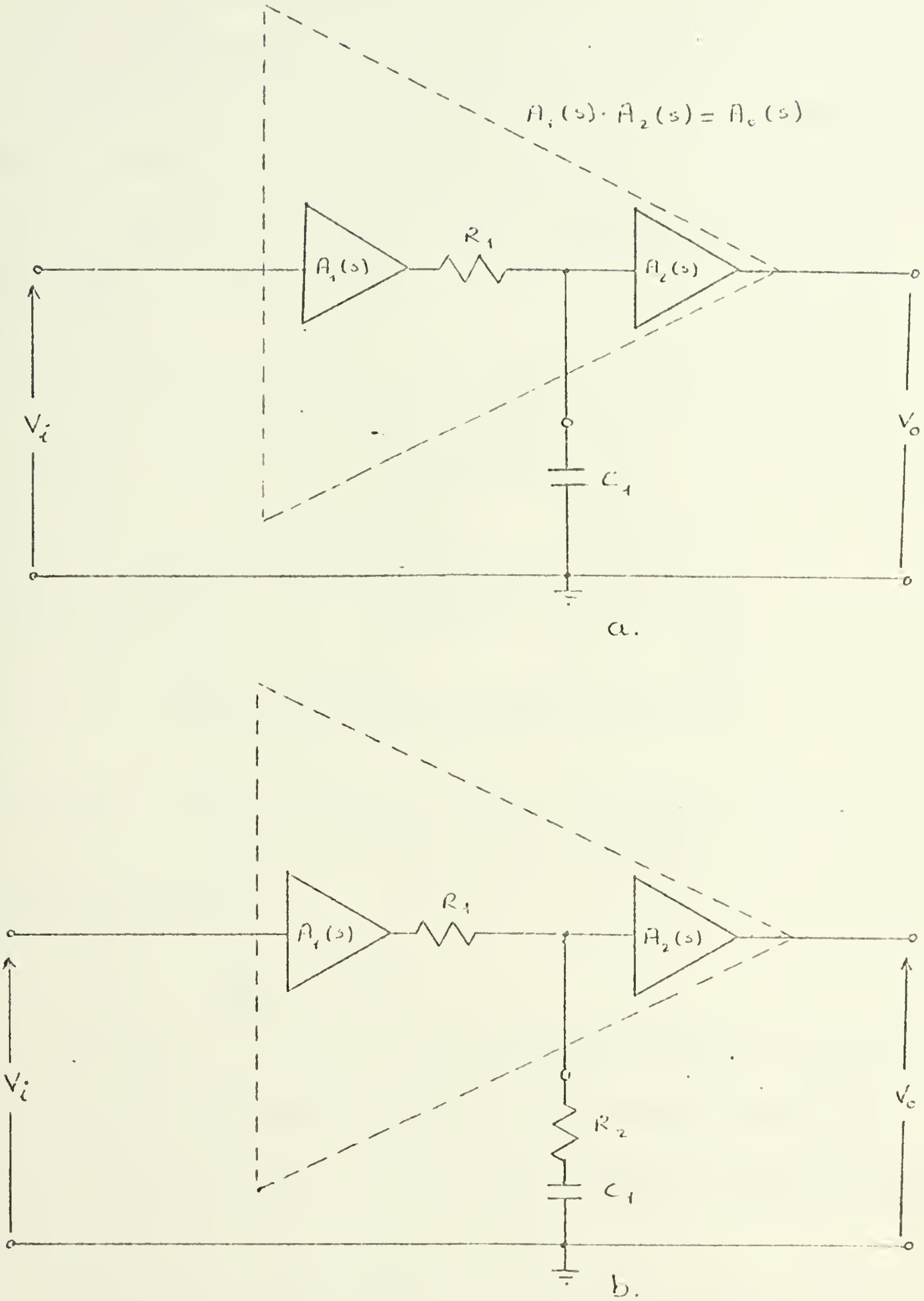


Figure II.19 Internal Compensation of Open Loop Gain.

Another method of compensation is external compensation, which consists of connecting an RC compensating network between output and input as shown in Fig. II.20. Here, the frequency response of the closed loop given is modified to ensure the stability. Consider for example the voltage inverter shown in Fig. II.20.

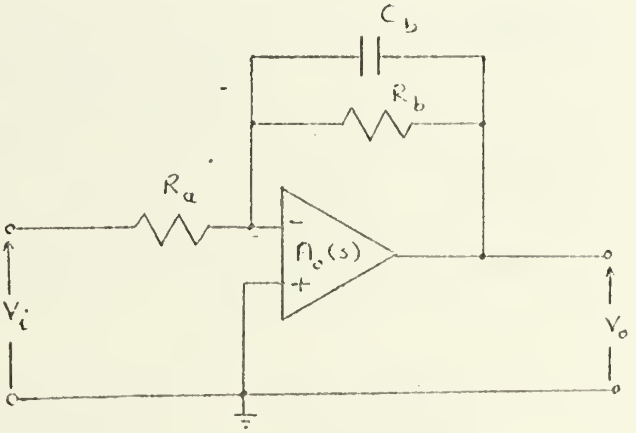


Figure II.20 Externally Compensated Voltage Inverter.

The closed loop gain is

$$A(s) = -\frac{R_b}{R_a} \cdot \frac{\omega_b}{(s+\omega_b)}$$

where $\omega_b = 1/R_bC_b$. If C_b is chosen such that the breakpoint associated with ω_b occurs before the crossover frequency $\omega_{o(\text{uncomp.})}$ then the rate of closure becomes 6 db/octave and a stable closed loop operation is obtained.

3. Fairchield μA741C IC - Operational Amplifier.

In the experimental part of this thesis the Fairchield μA741C integrated operational amplifier is used to realize the active RC networks. One reason for selecting this amplifier is that it is internally compensated by the manufacturer,

making it unnecessary to compensate by any of the previously mentioned methods. The specification sheet for the operational amplifier is presented in Appendix II. The open loop frequency response of the particular amplifier were measured with the setup shown in Fig. II.21, and they are found quite in agreement with the manufacturer's specification as shown in Fig. II.22.

4. Sensitivity Considerations in Networks with Operational Amplifiers

The relation between the sensitivity function and the pole and zero sensitivities of a network function has been mentioned in Section II-C which is now repeated:

$$S_k^{T(s)} = S_k^K - \sum_{i=1}^{m} \frac{S_k^{z_i}}{(s-z_i)} + \sum_{j=1}^{n} \frac{S_k^{p_i}}{(s-p_j)} \qquad \text{(II.14)}$$

It is well known that one of the major concerns in active RC network synthesis is that of minimizing the sensitivity function Eq. II.14.

One method of minimizing network sensitivity consists in judicially selecting the zeros (z_i) and poles (p_j) of the network function. To illustrate this consider the network function given below

$$T(s) = K\frac{N(s)}{D(s)} = \frac{K(s-z_1)(s-z_2)}{(s-p_1)(s-p_1^*)} = K\frac{s^2+a_1 s+a_0}{s^2+2\sigma s+\omega_1^2} \qquad \text{(II.15)}$$

The sensitivity of T(s) to a network parameter is according to Eqs. II.14 and II.15.

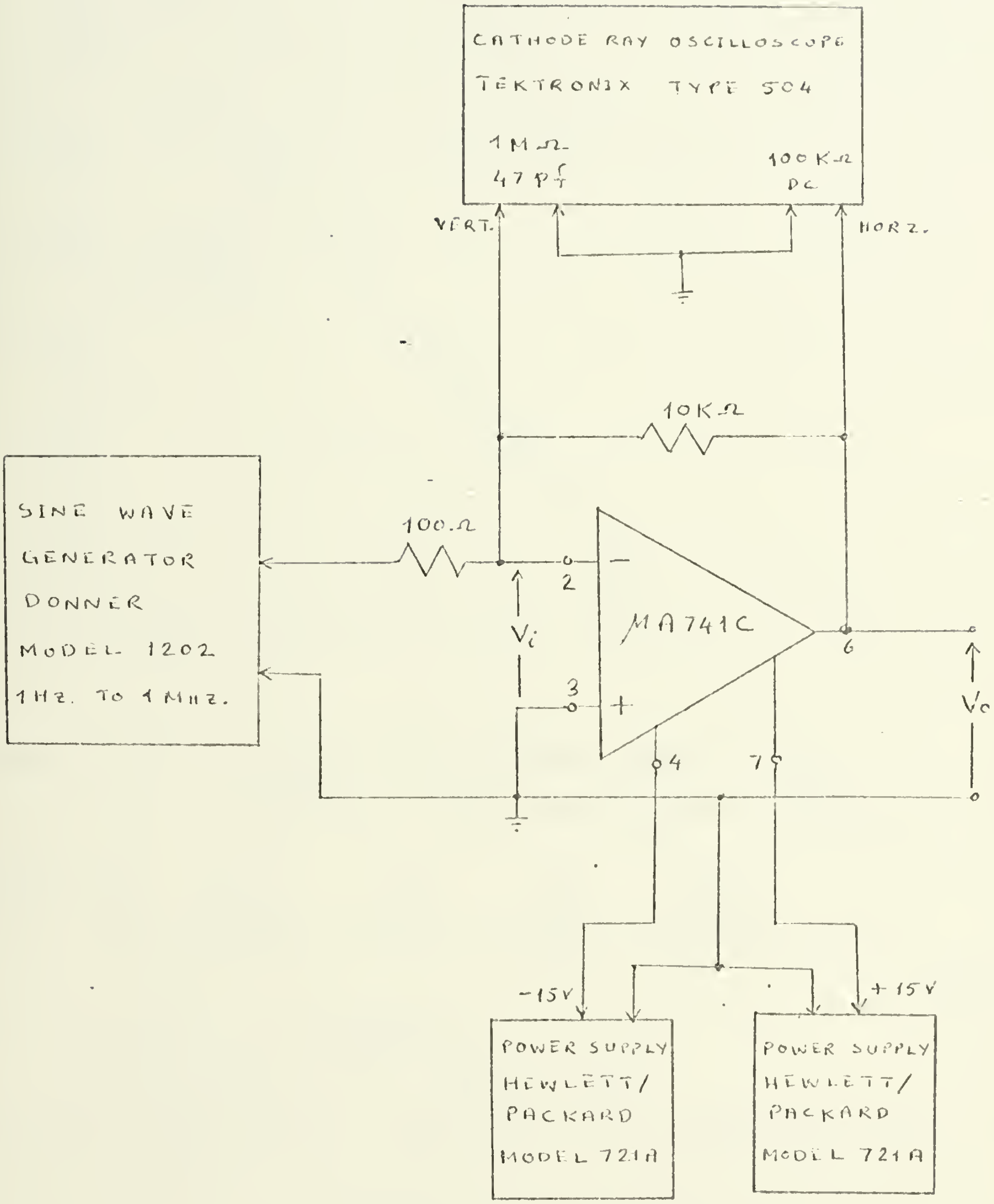


Figure II.21 Setup for Measuring the Open Loop Frequency Response.

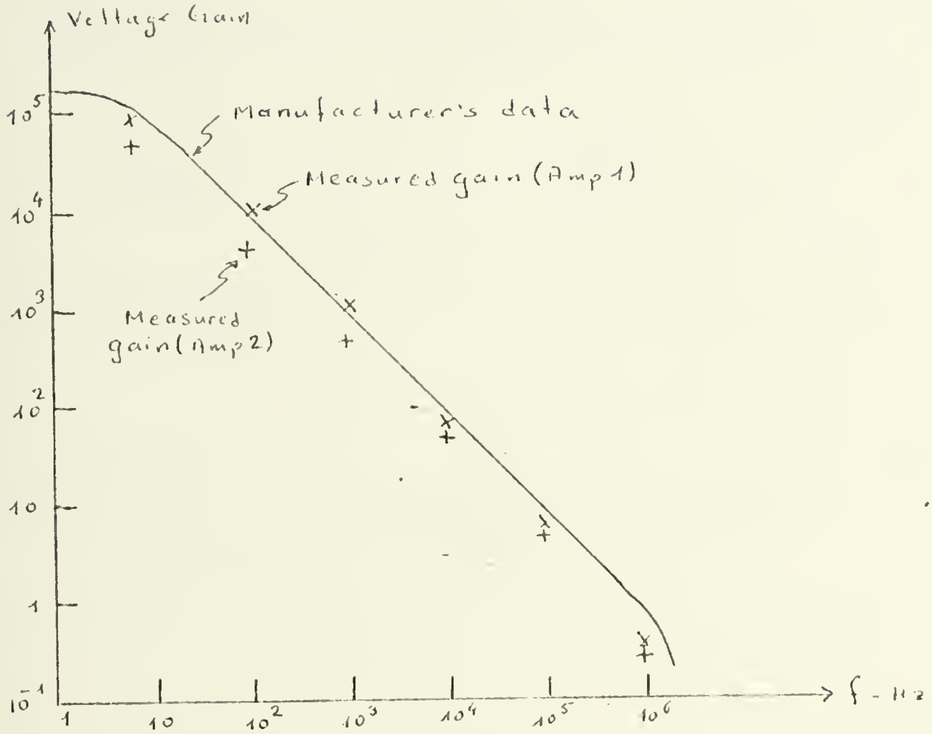


Figure II.22 The Open Loop Voltage Gain of the Particular μA741 Operational Amplifiers

$$S_k^{T(s)} = {}_{,}S_k^{K} - \frac{{}_{,}S_k^{z_1}}{(s-z_1)} - \frac{{}_{,}S_k^{z_2}}{(s-z_2)} + \frac{{}_{,}S_k^{p_1}}{(s-p_1)} + \frac{{}_{,}S_k^{p_1^*}}{(s-p_1^*)} \tag{II.16}$$

$$= \frac{A(s)}{(s-z_1)(s-z_2)(s-p_1)(s-p_1^*)}$$

where $A(s) = {}_{,}S_k^{K} N(s) D(s) - {}_{,}S_k^{z_1} D(s)(s-z_2) - {}_{,}S_k^{z_2} D(s)(s-z_1)$

$$+ {}_{,}S_k^{p_1} N(s)(s-p_1^*) + {}_{,}S_k^{p_1^*} N(s)(s-p_1)$$

In the neighborhood of pole frequency p_1 the term ${}_{,}S_k^{p_1}/(s-p_1)$ dominates and the sensitivity function can be approximated by

$$S_k^{T(s)} \simeq \frac{S_k^{p_1}}{(s-p_1)} \tag{II.17}$$

For $s = p_1$, Eqs. II.16 and II.17 can be equated.

$$S_k^{p_1} = (s-p_1)\, S_k^{T(s)} \Big|_{s=p_1} = \frac{A(p_1)}{(p_1-z_1)(p_1-z_2)\,2j\omega_1}$$

For a high Q filter $p_1 = -\sigma + j\omega_1 \simeq j\omega_1$

Thus

$$S_k^{T(j\omega)} \Big|_{s=j\omega_1} \simeq \frac{A\,p_1}{(p_1-z_1)(p_1-z_2)\cdot 2j\omega_1} \cdot \frac{1}{\sigma}$$

or

$$\left| S_k^{T(j\omega)} \right|_{s=j\omega_1} \simeq Q \cdot \frac{|A(p_1)|}{\omega_1^2 \left|(p_1-z_1)(p_1-z_2)\right|}$$

The following observations can be made about the above relation:

(1) Sensitivity function is proportional to Q hence it is desirable to obtain a transfer function with the lowest possible Q.

(2) Sensitivity function tends to be higher for low frequency applications because of the factor ω_1^2 in the denominator.

(3) The zeros z_1 and z_2 should be as far as possible from the pole p_1, because this maximizes $|(p_1-z_1)(p_1-z_2)|$ and so minimizes $S_k^{T(j\omega)}$. However this freedom of choosing z_1, z_2 and p_1 usually does not exist, because

the poles and zeros are determined by the desired circuit function T(s). In view of Eq. II.14 and these remarks, the pole and zero sensitivities should be minimized independently. In particular, pole sensitivities should be minimized to ensure stability.

The pole sensitivity with respect to a variable parameter is

$$S_k^{p_j} = \frac{dp_j}{dk/k}$$

Hence the drift of j^{th} pole due to the variation of k is

$$dp_j = S_k^{p_j} \cdot \frac{dk}{k}$$

The total drift of j^{th} pole due to variations of all network parameter is

$$dp_j = \sum_{i=1}^{r} S_{R_i}^{p_j} \cdot \frac{dR_i}{R_i} + \sum_{i=1}^{c} S_{C_i}^{p_j} \cdot \frac{dC_i}{C_i} + \sum_{i=1}^{a} S_{A_i}^{p_j} \cdot \frac{dA_i}{A_i} \qquad \text{(II.18)}$$

where r is the total number of resistors R_i, where c is the total number of capacitors c_i, and where a is the total number of active elements A_i in the network. The effect of active element gain A_i can be lumped into one equivalent active device A. Since integrated circuits have very good component tracking capabilities it is valid to assume that the passive element variations are uniform. Thus Eq. II.18 can be rewritten as

$$dp_j = \frac{dA}{A} \cdot S_A^{p_j} + \frac{dR}{R} \sum_{i=1}^{r} S_{R_i}^{p_j} + \frac{dC}{C} \sum_{i=1}^{c} S_{C_i}^{p_j}$$

where $\frac{dR}{R} = \frac{dR_i}{R_i}$ (i=1,2,⋯,r) and similarly for C. Three new functions are now defined

$$S_A = \frac{dp_j/p_j}{dA/A} = \frac{S_A^{p_j}}{p_j}, \qquad S_{R_i} = \frac{dp_j/p_j}{dR_i/R_i} = \frac{S_{R_i}^{p_j}}{p_j}$$

$$S_{C_i} = \frac{dp_j/p_j}{dC_i/C_i} = \frac{S_{C_i}^{p_j}}{p_j}.$$

Then:

$$\frac{dp_j}{p_j} = S_A\frac{dA}{A} + \frac{dR}{R}\cdot\sum_{i=1}^{r} S_{R_i} + \frac{dC}{C}\sum_{i=1}^{c} S_{C_i} \tag{II.19}$$

It is possible to calculate the term $\sum_{i=1}^{c} S_{C_i}$ by performing frequency scaling on the network, without changing its admittance level, such that $\omega' = a\omega$. The new pole location p_j' is $p_j' = ap_j$. Because A and R are not functions of ω, they do not change, hence dA/A = dR/R = 0 when ω changes to ω'. However the admittance of C changes from its old value of ωC to its new value of aωC. In order to keep the admittance level of C unchanged, a new capacitor value $C' = C/a$ is needed. The values

$$\left.\frac{dp_j'}{p_j'}\right|_{p_j} = \frac{da}{a}, \qquad \left.\frac{dC'}{C'}\right|_{C} = -\frac{da}{a}, \qquad \frac{dA}{A} = \frac{dR}{R} = 0$$

are substituted into Eq. II.19 yielding $\frac{da}{a} = -\frac{da}{a}\sum_{i=1}^{c} S_{C_i}$

and so $\sum_{i=1}^{c} S_{C_i} = -1$

Then Eq. II.19 is rewritten

$$\frac{dp_j}{p_j} = S_A \cdot \frac{dA}{A} - \frac{dC}{C} + \frac{dR}{R} \cdot \sum_{i=1}^{r} S_{R_i} \qquad \text{(II.20)}$$

It is also possible to calculate the term $\sum_{i=1}^{r} S_{R_i}$ by performing admittance scaling on the network without changing the frequency level, such that $Y_R' = a Y_R$, $Y_c' = a Y_c$. $p_j' = p_j$ ($dp_j = 0$) , $\omega' = \omega$. Because A has no dimension dA/A=0 for admittance scaling. $Y_R' = a Y_R = a/R = 1/R'$ hence $R' = R/a$. $Y_c' = a Y_c = \omega a C = \omega C'$ hence $C' = aC$. The values

$$\left.\frac{dC'}{C'}\right|_C = \frac{da}{a} \ , \ \left.\frac{dR'}{R'}\right|_R = -\frac{da}{a} \ , \ \frac{dA}{A} = \frac{dp_j}{p_j} = 0$$

are substituted into Eq. II.20 yielding

$$\sum_{i=1}^{r} S_{R_i} = -1 \ .$$

Then Eq. II.20 is rewritten

$$\frac{dp_j}{p_j} = \frac{dA}{A} S_A - \left(\frac{dR}{R} + \frac{dC}{C} \right)$$

By substituting for S_A , the pole drift is obtained

$$dp_j = S_A^{p_j} \frac{dA}{A} - p_j \left(\frac{dR}{R} + \frac{dC}{C} \right) \qquad \text{(II.21)}$$

From Eq. II.21 the direction of the change in pole locations due to gain A or passive elements (R, C) variations can be found. Assume $(\frac{dR}{R} + \frac{dC}{C}) = 0$, and so

$dp_j = S_A^{p_j} \frac{dA}{A}$. Because dA/A is a real number for low frequencies

$$\angle dp_j = \angle S_A^{p_j} \tag{II.22}$$

Eq. II.22 implies that the directional variation of pole location dp_j due to the variation of A is equal to the argument of the sensitivity function with respect to A. On the other hand assume dA/A = 0, when $dp_j = -p_j \left(\frac{dR}{R} + \frac{dC}{C}\right)$. Then $\angle dp_j = \angle p_j + \pi$ because $\left(\frac{dR}{R} + \frac{dC}{C}\right)$ is a positive real number. This implies that the directional variation of pole location due to the passive element changes are in a direction radial with the origin. If the pole sensitivity is minimized the equality

$$S_A^{p_j} \cdot \frac{dA}{A} - p_j \left(\frac{dR}{R} + \frac{dC}{C}\right) = 0 \tag{II.23}$$

must be satisified.

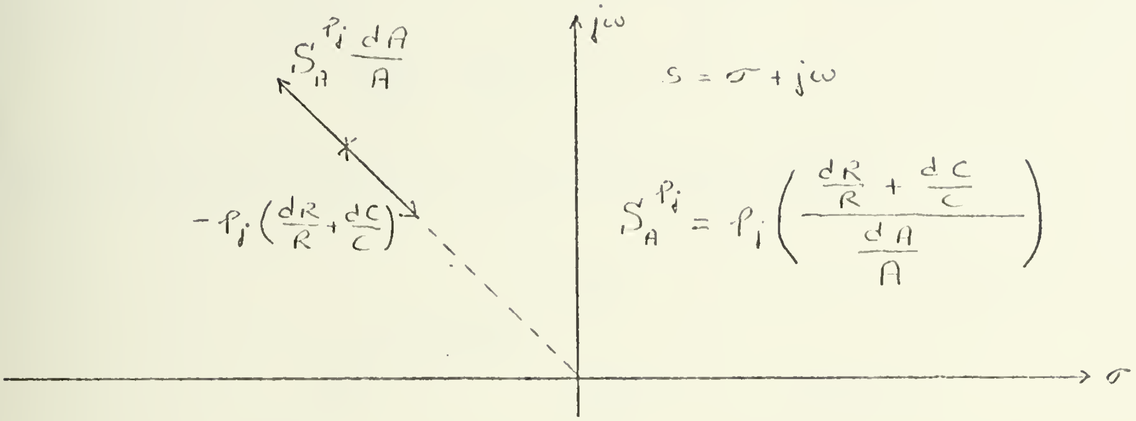
a. Pole Desensitization

Eq. II.23 provides three techniques for pole desensitization [38].

(1) *Technique 1.* Pole desensitization is obtained by designing the network in such a way that the passive pole displacement is compensated by the active pole displacement [39]. That is

$$S_A^{p_j} = p_j \left(\frac{dR}{R} + \frac{dC}{C}\right) \cdot \frac{A}{dA} \tag{II.24}$$

This method is shown in Fig. II.23.



$S_A^{p_i} \frac{dA}{A}$
$-p_i\left(\frac{dR}{R}+\frac{dC}{C}\right)$
$j\omega$
$s = \sigma + j\omega$
$S_A^{p_i} = p_i\left(\frac{\frac{dR}{R}+\frac{dC}{C}}{\frac{dA}{A}}\right)$
σ

Figure II.23 Pole Desensitization by Technique 1

This technique is somewhat complicated since the chosen feedback configuration depends both on the physical properties of the active and passive components being used and on the location of the particular pole being desensitized. The requirement imposed by Eq. II.24 is restrictive enough to limit severely the choice of network configurations capable of satisfying both Eq. II.24 and the characteristics of the desired network function simultaneously. This method is not suitable for network synthesis with operational amplifiers since the gain characteristics of the amplifiers must be individually controllable or variable with ambient variations which they usually are not.

(2) Technique 2. In the second desensitization technique, the network configurations used are those, whose critical pole sensitivities depend only on the passive network elements. For some structures the pole sensitivity with respect to the active elements can be made arbitrarily small.

Then the poles are desensitized by using resistors and capacitors with uniformly equal but opposite temperature coefficients. Hence

$$S_A^{p_i} \simeq 0 \quad , \qquad \frac{dR}{R} + \frac{dC}{C} \simeq 0$$

This desensitization is shown in Fig. II.24

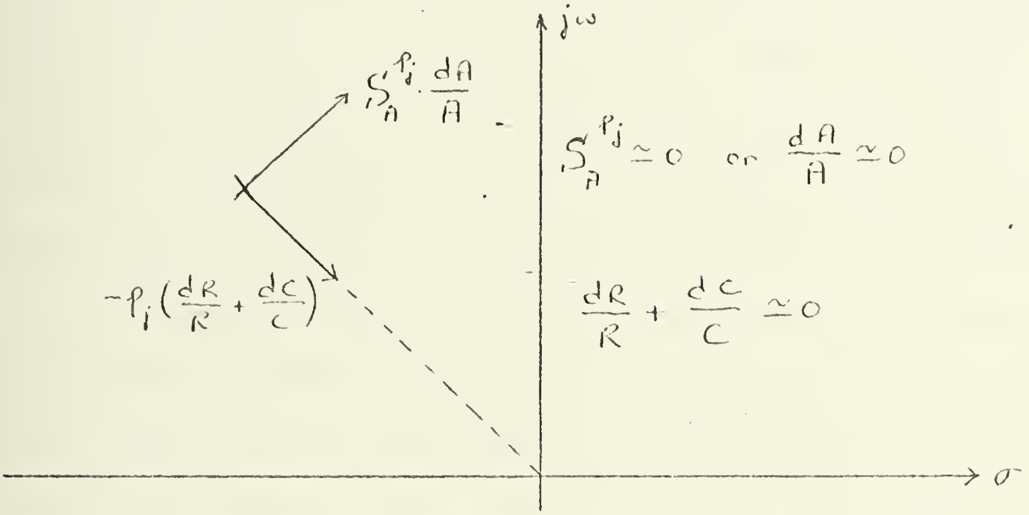

$j\omega$
$S_A^{p_i} \cdot \frac{dA}{A}$
$S_A^{p_j} \simeq 0$ or $\frac{dA}{A} \simeq 0$
$-p_i\left(\frac{dR}{R} + \frac{dC}{C}\right)$
$\frac{dR}{R} + \frac{dC}{C} \simeq 0$
σ

Figure II.24 Pole Desensitization by Techniques 2 and 3.

Typical network configurations for this technique are the negative - feedback configurations, and unity gain forward feedback configurations [38]. In both cases the pole sensitivity to A can be made arbitrarily small by the use of amplifiers with high loop gain. Thus operational amplifiers are ideal for this technique.

(3) Technique 3. In this technique the pole sensitivities are dependent on both active and passive components as in the technique 1. The sensitivity with respect to passive element is again minimized by using resistors and capacitors with uniformly equal but opposite drift characteristics. However in this technique the sensitivity with respect

to active element is not minimized, but the drift of the active element gain is minimized. Hence

$$\frac{dA}{A} \simeq 0 \quad , \quad \frac{dR}{R} + \frac{dC}{C} = 0$$

The desensitization scheme is same as shown in Fig. II.24.

The active element can be stabilized by a local negative feedback network consisting of passive components with good tracking properties. Since the stabilization is much more effective with high open loop gain of the active element, the operational amplifiers are also ideal for this technique. But since available closed loop gain of an amplifier is greatly reduced by local stabilizing network, in high Q applications more than one amplifier may be required. Since there is no limitation on pole sensitivity with respect to active elements, this technique gives more freedom for choosing the network configuration to be used.

III. INFINITE GAIN SYNTHESIS TECHNIQUES

A. INTRODUCTION

In the preceding chapters, the advantages of using operational amplifiers in the synthesis of active RC networks were indicated and the theory of operational amplifiers was discussed. Particular mention was made of the μA741C operational amplifier.

In the past ten years a great many synthesis techniques with operational amplifiers have been proposed and experimental results are given in the literature. The number of proposed circuits is quite large and there is need to classify them and for a common theory. The purpose of this thesis is to give such a classification and evaluation based on a common theory. Circuits are classified with respect to the number of amplifiers in the circuit. For the single operational amplifier circuits a subclassification has been made with respect to the number of feedback paths.

B. SINGLE OPERATIONAL AMPLIFIER SYNTHESIS

1. General

In the active RC network synthesis with a single operational amplifier, the most general circuit configuration is shown in Fig. III.1. The RC network in Fig. III.1 can be considered to be a six terminal feedback network. The input signals to the RC network are V_1 and V_2. The feedback signal to the RC network is V_o and the output signals are V_3 and V_4. The feedback network can be a single network terminated in the negative input of the operational amplifier or it can

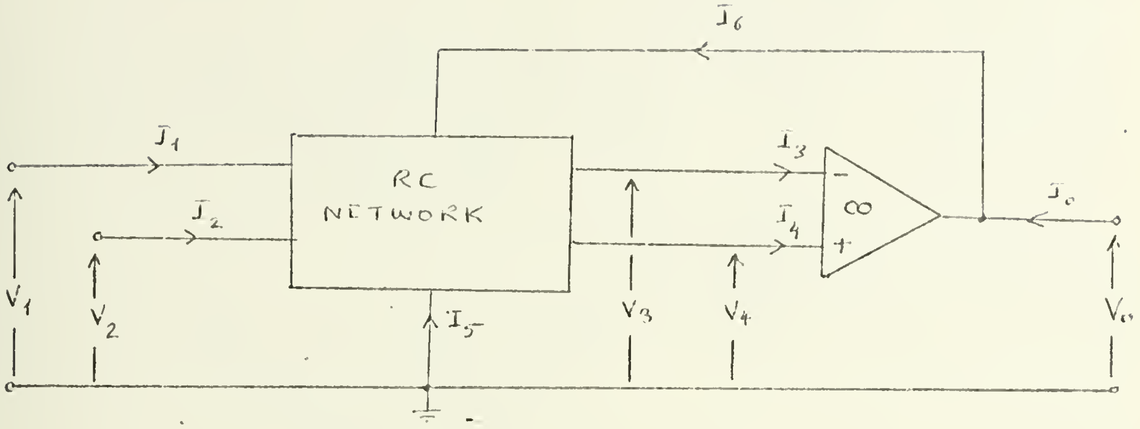


Figure III.1 The General Configuration of Single Operational Amplifier Active RC Circuits

consist of a number of networks terminated in both inputs. It is appropriate to separate the synthesis problem into two parts.

a. Single feedback synthesis.

b. Multiple feedback synthesis.

These methods will now be discussed.

2. Single Feedback Synthesis

The general circuit configuration for a single feedback network based on Fig. III.1 can be shown as in Fig. III.2. In general, the two input circuits and one feedback circuit might be active RC, or passive RC circuits. However since the single operational amplifier realizations is considered, it is convenient to make these networks passive RC two ports.

The input and output relations of these RC networks can be expressed by their two port admittance parameters as below:

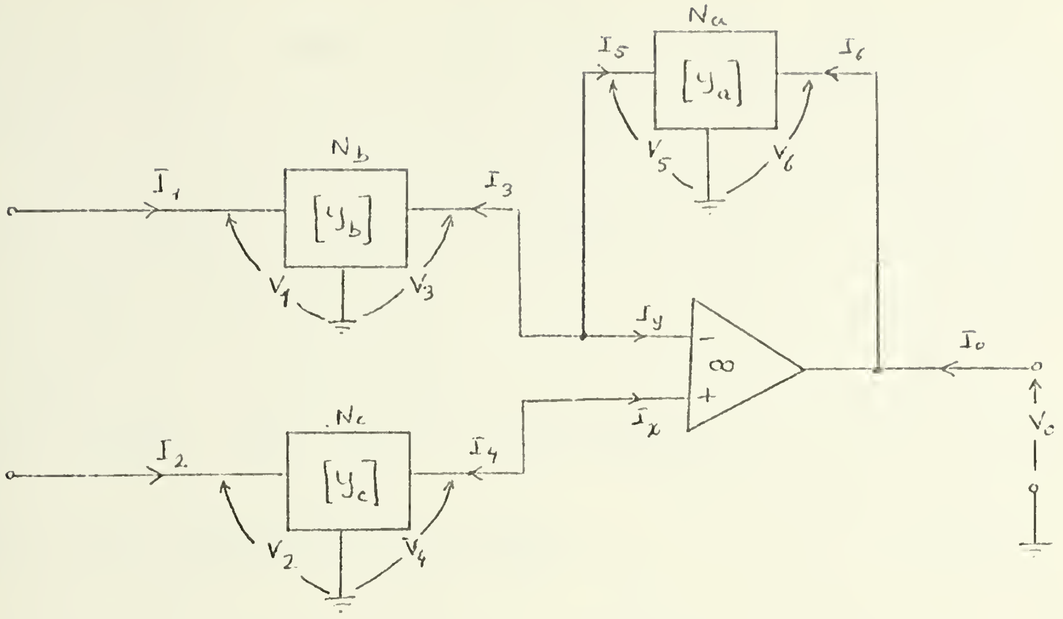


Figure III.2 The General Single Feedback Operational Amplifier Circuit.

$$I_1 = y_{11b}V_1 + y_{12b}V_3 \tag{III.1a}$$

$$I_3 = y_{21b}V_1 + y_{22b}V_3 \tag{III.1b}$$

$$I_2 = y_{11c}V_2 + y_{12c}V_4 \tag{III.2a}$$

$$I_4 = y_{21c}V_2 + y_{22c}V_4 \tag{III.2b}$$

$$I_5 = y_{11a}V_5 + y_{12a}V_6 \tag{III.3a}$$

$$I_6 = y_{21a}V_5 + y_{22a}V_6 \tag{III.3b}$$

Since the input impedance of an ideal operational amplifier is infinite,

$$I_x = I_4 = 0 \quad , \quad I_y = 0$$

Hence from Eq. III.2b

$$V_4 = -V_2\left(\frac{y_{21c}}{y_{22c}}\right) \tag{III.4}$$

and also $I_3 = -I_5$. Then by equating the Eqs. III.1b and III.3a

$$y_{21b} V_1 + y_{22b} V_3 = -y_{11a} V_5 - y_{12a} V_6 \qquad \text{(III.5)}$$

As can be seen from Fig. III.2

$$V_3 = V_5 \quad \text{and} \quad V_6 = V_0$$

Therefore the Eq. III.5 can be rewritten as

$$V_0 = -\left(\frac{y_{21b}}{y_{12a}}\right) V_1 - \left(\frac{y_{11a} + y_{22b}}{y_{12a}}\right) V_3 \qquad \text{(III.6)}$$

Since tha gain of the operational amplifier is ideally infinite $V_3 - V_4 = 0$, and with Eq. III.5

$$V_3 = V_4 = -\left(\frac{y_{21c}}{y_{22c}}\right) V_2$$

Then by substituting the above into Eq. III.6

$$V_0 = -\left(\frac{y_{21b}}{y_{12a}}\right) V_1 + \left(\frac{y_{11a} + y_{22b}}{y_{12a}}\right) \cdot \left(\frac{y_{21c}}{y_{22c}}\right) V_2 \qquad \text{(III.7)}$$

By making y_{22c} infinite, the second term of the Eq. III.7 can be removed. Then the voltage transfer function of the circuit becomes

$$T(s) = \frac{V_0}{V_1} = -\frac{y_{21b}}{y_{12a}} \qquad \text{(III.8)}$$

After making y_{22c} infinite, the final circuit configuration is shown in Fig. III.3.

Since the input and the feedback networks consist of linear, lumped, passive RC elements, the transfer function Eqn. III.8 is a rational polynomial in s. For a given voltage transfer function T(s) = Q(s)/N(s) to be realized,

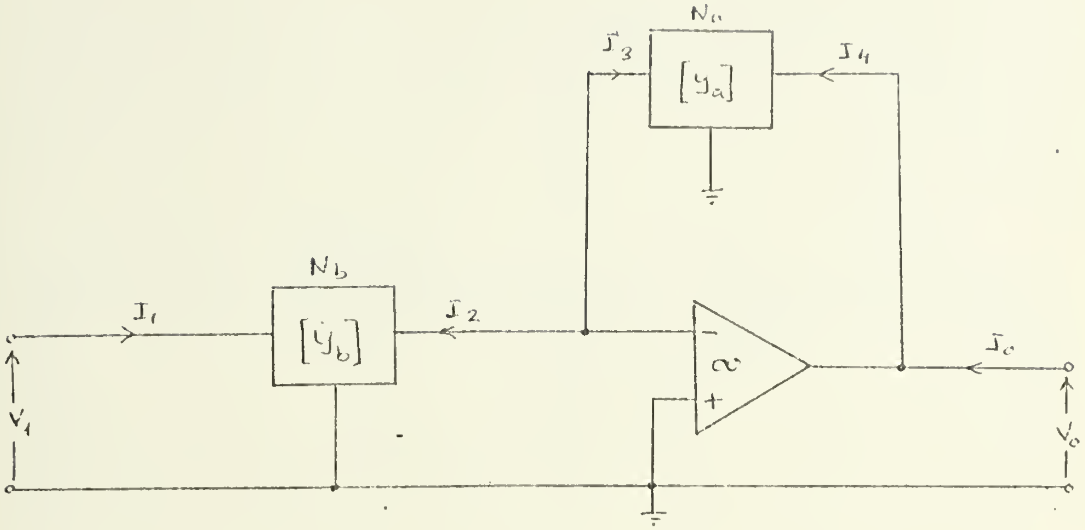


Figure III.3 The Final Single Feedback Circuit

Q(s) is assigned as the numerator polynomial of $-y_{21b}(s)$ and N(s) is assigned as the numerator polynomial of $-y_{12a}(s)$. Then the passive RC network synthesis techniques discussed in the second chapter can be used.

The above technique is one of the earliest and most frequently used methods in active RC network synthesis using one operational amplifier [2,14,26,40,41,42].

a. Sensitivity Considerations

In the foregoing development of the transfer function of single feedback synthesis the ideal operational amplifier characteristics were assumed. Now the effect of the finite gain and finite input impedance will be investigated. For this purpose, a more realistic network configuration is given in Fig. III.4. In this figure the amplifier is assumed to have a finite gain of $A_o(s)$ and the finite input admittance y_i has been added external to the

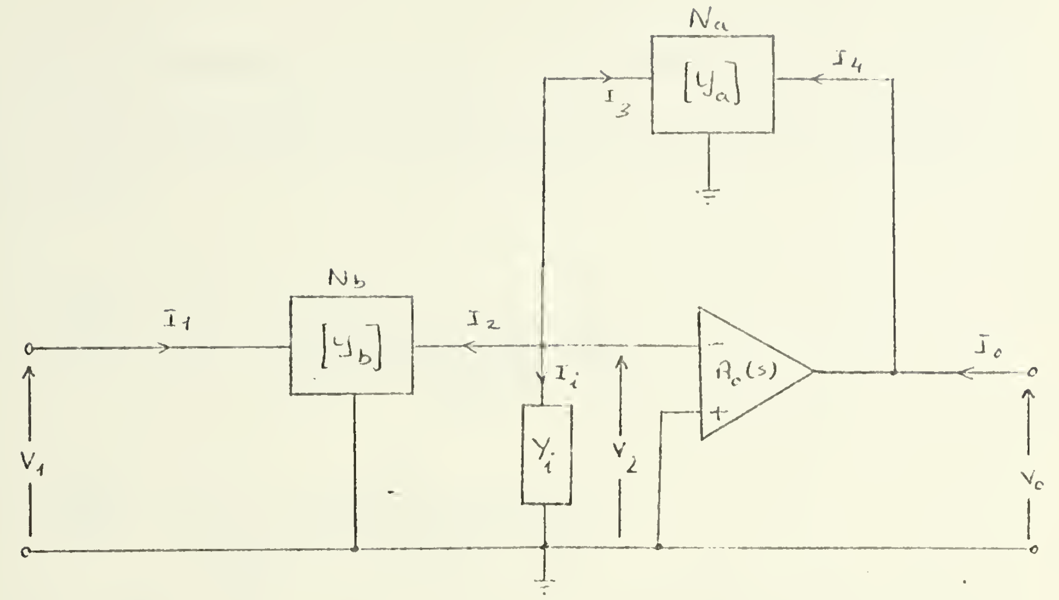


Figure III.4 Single Feedback with a Nonideal Operational Amplifier.

amplifier. Since the analysis is made with no load the output impedance can be included in network, N_a.

For Fig. III.4 in addition to the two port admittance relations of N_a and N_b, the following relations are valid.

$$I_i = V_2 Y_i \quad , \quad I_2 + I_3 + I_i = 0 \quad , \quad V_o = -A_o(s) V_2$$

The corresponding voltage transfer function is

$$\frac{V_o}{V_1} = -\frac{y_{21b}}{y_{12a} - \frac{1}{A_o(s)}\left[y_{22b} + y_{11a} + Y_i\right]} \qquad \text{(III.9)}$$

The term with $A_o(s)$ in the denominator above represents an error term due to the finite gain and the input impedance of the operational amplifier. Because the parameters y_{11a} and y_{22b} are not known until the synthesis procedure is complete, it is not possible to predict and

eliminate this error at the start. This is one disadvantage of the single feedback technique. The error can be reduced by judiciously selecting the y_{22b}, y_{11a} as will be seen now. From Eq. III.9

$$\frac{V_o}{V_1} = -\frac{y_{21b}}{y_{12a}} \cdot \frac{1}{1 - \frac{1}{A_o(s)}\left[\frac{y_{22b} + y_{11a} + Y_i}{y_{12a}}\right]}$$

$$= -\frac{Q(s)}{N(s)} \cdot \frac{1}{1 - \frac{1}{A_o(s)}\left[\frac{2P(s) + Y_i/D(s)}{N(s)}\right]}$$

where Q(s), N(s), P(s) and D(s) are the polynomials defined in three terminal RC synthesis techniques and were mentioned in Section II.A. The error term must be kept small near the zero frequencies of the polynomial N(s). Thus if the roots of P(s) are chosen near to the roots of N(s) the error becomes negligible.

As has been mentioned before it is not possible to know the error at the start of the synthesis procedure. Further, it is not possible to investigate the pole or Q sensitivities of the polynomial N(s) when the unknown error is present. However the sensitivity of $T(s) = V_o(s)/V_1(s)$ can be found with respect to amplifier gain or admittance polynomials Q(s), N(s), P(s) and D(s) as follows:

$$S^{T(s)}_{A_o(s)} = \frac{1}{1 - \left[\frac{A_o(s)\,N(s)}{2P(s) + Y_i/D(s)}\right]}$$

$$S^{T(s)}_{Q(s)} = -1$$

$$S_{N(s)}^{T(s)} = \frac{1}{1 - \left[\frac{2P(s) + Y_i / D(s)}{A_c(s)\, N(s)}\right]}$$

$$S_{P(s)}^{T(s)} = - \frac{2P(s)}{A_c(s)\, N(s) - \left[\frac{Y_i}{D(s)} + 2\, P(s)\right]}$$

$$S_{D(s)}^{T(s)} = \frac{1}{1 - D(s)\left[\frac{A_c(s)\, N(s) - 2\, P(s)}{Y_i}\right]}.$$

All of the sensitivities are complex functions of frequency and can be numerically calculated after the synthesis is completed.

3. Multiple Feedback Synthesis

a. General

In the single operational amplifier multiple feedback synthesis the output of the operational amplifier is fed to the input by means of RC two terminal elements as shown for example in Fig. III.5.

All multiple feedback realizations can be classified according to three synthesis techniques. Double-Ladder synthesis, single ladder synthesis, and two special circuits which were proposed by Brugler [21] and Bohn [43].

b. Double-Ladder Synthesis

This synthesis technique covers a great many of the active RC networks found in the literature. For example, second order realizations with double-ladder technique were discussed to some extent by Bridgeman and Brennan [44].

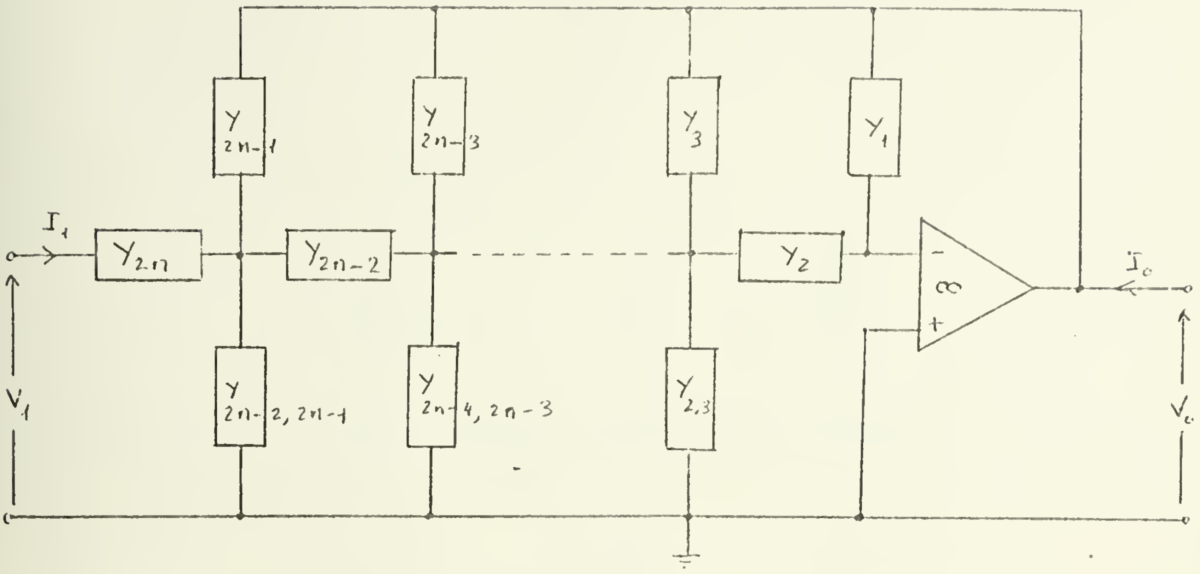


Figure III.5 Double-Ladder Configuration

Wadhwa's [45] circuit for third order system is one special case of this technique. Rauch's [46] filter is another special case and even some of the Sallen-Key [33] structures can be mentioned as special cases of this technique.

The general circuit configuration of double-ladder synthesis is shown in Fig. III.5. In this figure the admittances are one-port networks which may consist of one or more R or C elements. For the open circuit voltage transfer functions of this synthesis, an explicit algorithm has been given by Holt and Sewell [47]. However, a general way of finding the voltage transfer function of any circuit having one or more operational amplifiers has been established by Nathan [48]. His method is used throughout the multiple feedback synthesis and it is discussed in Appendix I. In the calculations of error due to the finite gain and finite input impedance of the amplifier, Nathan's [49] method for finite gain amplifier circuits has been used.

The double-ladder synthesis is illustrated by a general example which has a circuit configuration shown in Fig. III.6

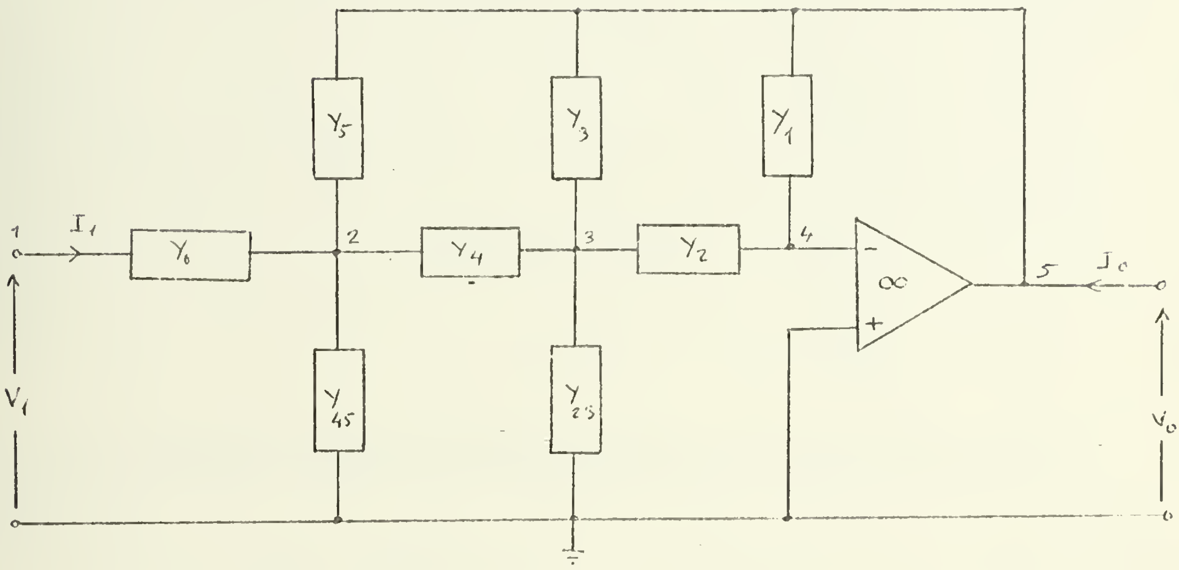


Figure III.6 A General Example of Double-Ladder Synthesis

The admittance matrix of the passive part of the circuit, with the operational amplifier removed is where

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & Y_6 & -Y_6 & 0 & 0 & 0 \\ 2 & -Y_6 & (Y_4+Y_5+Y_6+Y_{45}) & -Y_4 & 0 & -Y_5 \\ 3 & 0 & -Y_4 & (Y_2+Y_3+Y_4+Y_{23}) & -Y_2 & -Y_3 \\ 4 & 0 & 0 & -Y_2 & (Y_1+Y_2) & -Y_1 \\ 5 & 0 & -Y_5 & -Y_3 & -Y_1 & (Y_1+Y_3+Y_5) \end{array}$$

terminal 6 has been grounded. By deleting the row of the driven node 5 and the column of the driving node 4, the new

admittance matrix for the overall circuit with the operational amplifier is found.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & 5 \\ \hline 1 & Y_6 & -Y_6 & 0 & 0 \\ 2 & -Y_6 & (Y_4+Y_5+Y_6+Y_{45}) & -Y_4 & -Y_5 \\ 3 & 0 & -Y_4 & (Y_2+Y_3+Y_4+Y_{23}) & -Y_3 \\ 4 & 0 & 0 & -Y_2 & -Y_1 \end{array}$$

Then the voltage transfer function of the circuit in Fig. III.6 is

$$\frac{V_o}{V_i} = \frac{Y^{15}}{Y^{11}}$$

where the superscripts denote the cofactors. This transfer function is

$$\frac{V_o}{V_i} = \frac{Y_2 Y_4 Y_6}{Y_1\left[(Y_2+Y_3+Y_4+Y_{23})(Y_5+Y_6+Y_{45})+Y_4(Y_2+Y_3+Y_{23})\right] + Y_2\left[Y_3(Y_4+Y_5+Y_6+Y_{45})+Y_4 Y_5\right]} \tag{III.10}$$

Assume that it is desired to realize a transfer function such as

$$T(s) = \frac{b s^2}{a_3 s^3 + a_2 s^2 + a_1 s + 1} \tag{III.11}$$

By comparing Eq. III.10 and Eq. III.11, it can be seen that of the admittances y_2, y_4, y_6, two should be capacitive,

while the third should be conductive. Therefore one selection of these three admittances is as follows

$$Y_2 = C_2 s\,, \quad Y_4 = C_4 s\,, \quad Y_6 = G_6$$

Since the denominator of the Eq. III.11 contains a third order term, another admittance must be capacitive. Hence

$$Y_3 = C_3 s$$

The rest of the circuit consists of conductanctances. With these components the coefficients of the desired transfer function are found as

$$b = \frac{C_2 C_4 G_6}{G_1 G_{23} (G_5 + G_6 + G_{45})}$$

$$a_1 = \frac{C_2 + C_3 + C_4}{G_{23}} + \frac{C_4}{G_5 + G_6 + G_{45}}$$

$$a_2 = \frac{C_2 C_3}{G_1 G_{23}} + \frac{C_4 (C_2 + C_3)}{G_{23}(G_5 + G_6 + G_{45})} + \frac{C_2 C_4 G_5}{G_1 G_{23} (G_5 + G_6 + G_{45})}$$

$$a_3 = \frac{C_2 C_3 C_4}{G_1 G_{23} (G_5 + G_6 + G_{45})}$$

Therefore the circuit which realizes the transfer function of Eq. III.11 is as shown in Fig. III.7.

In the circuit shown in Fig. III.7 eight two-terminal elements are used as admittance networks. However this is not necessary. As an example the same transfer function is synthesized with a simpler configuration as shown in Fig. III.8. By the same method employed above, the voltage transfer function of this circuit is found.

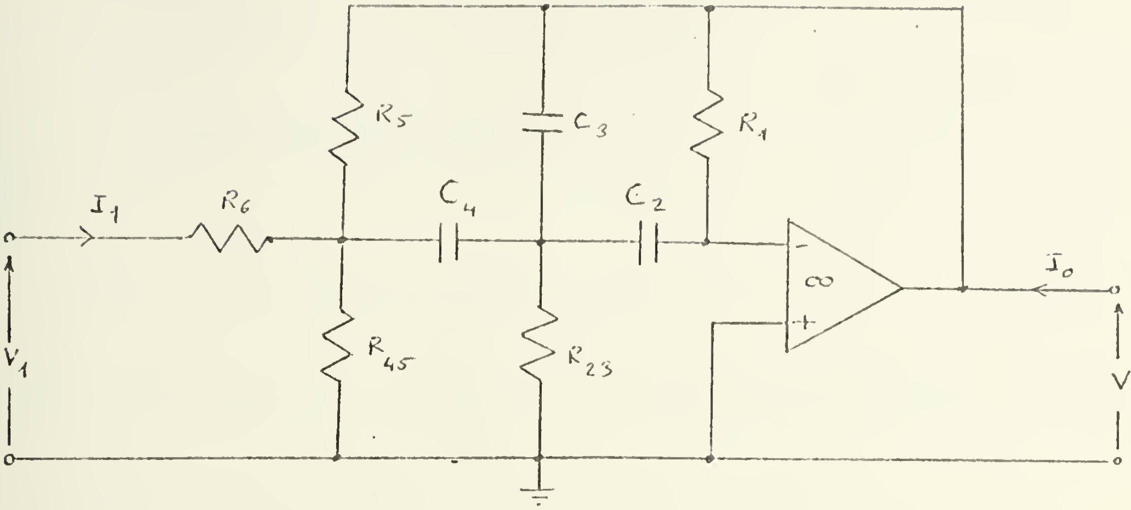


Figure III.7 Final Circuit of the Example

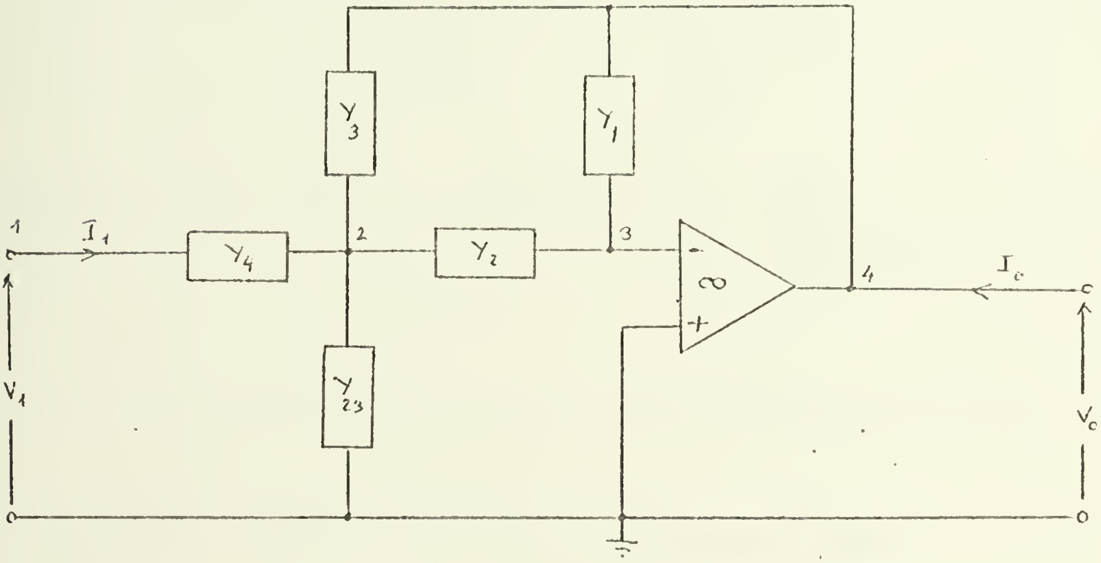


Figure III.8 Second Example of Double Ladder Synthesis

$$\frac{V_o}{V_1} = -\frac{Y_2 Y_4}{Y_1(Y_2+Y_3+Y_4+Y_{23}) + Y_2 Y_3}$$

and by selecting

$$Y_2 = \frac{C_2 s}{R_2 C_2 s + 1}, \quad Y_4 = \frac{C_4 s}{R_4 C_4 s + 1}, \quad Y_3 = C_3 s$$

and the rest of the elements as conductances the voltage transfer function of the circuit becomes

$$T(s) = -\frac{b\,s^2}{a_3 s^3 + a_2 s^2 + a_1 s + 1}$$

where

$$a_1 = R_{23}(C_2 + C_3 + C_4) + R_2 C_2 + R_4 C_4$$

$$a_2 = C_2 C_4 (R_{23} R_4 + R_{23} R_2 + R_2 R_4) + C_2 C_3 R_{23} (R_1 + R_2) + R_{23} R_4 C_3 C_4$$

$$a_3 = R_{23} R_4 C_2 C_3 C_4 (R_1 + R_2) \; , \quad b = R_1 R_{23} C_2 C_4$$

The final circuit is shown in Fig. III.9

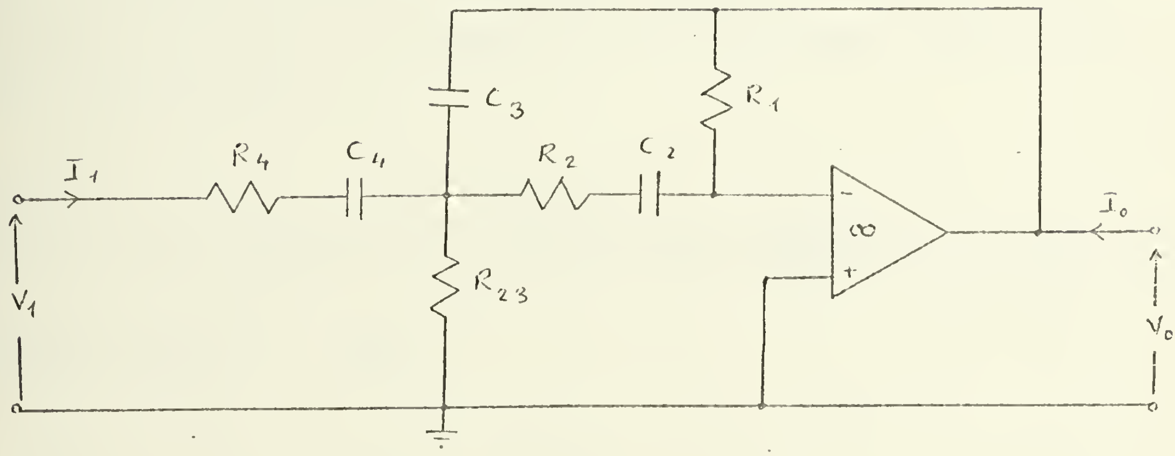


Figure III.9 Final Circuit of the Second Example

Note that the number of elements remains the same but that one circuit may give better element values than the other.

(1) Sensitivity Considerations. Double-ladder synthesis has one advantage over the single feedback synthesis, namely, the effect of the finite gain, input and output admittances can be accounted for and can be corrected at the design

stage. Consider a given second order transfer function

$$T(s) = \frac{Ks}{s^2 + 2\sigma s + \omega_n^2} \qquad \text{(III.12)}$$

T(s) is to be realized with the circuit shown in Fig. III.8, which has a voltage transfer function

$$\frac{V_o}{V_1} = \frac{Y_2 Y_4}{Y_1(Y_2 + Y_3 + Y_4 + Y_{23}) + Y_2 Y_3} \qquad \text{(III.13)}$$

By comparing Eqs. III.12 and III.13 an initial assignment for the element values may be made as follows:

$$Y_2 = C_2 s\,, \quad Y_3 = C_3 s\,, \quad Y_1 = G_1\,, \quad Y_4 = G_4\,, \quad Y_{23} = G_{23}$$

Now assume the amplifier has a finite gain A_o and a finite input admittance Y_i. Then the element values in Fig. III.8 can be amended and the resulting transfer function can be found (by Nathan's [49] method for finite gain amplifiers). The new function is

$$\frac{V_o}{V_1} = \frac{Y_2 Y_4}{\left(1 + \frac{1}{A_o}\right)\left[Y_1(Y_2 + Y_3 + Y_4 + Y_{23}) + Y_2 Y_3\right] + \frac{1}{A_o}\left[Y_i(Y_2 + Y_3 + Y_4 + Y_{23}) + Y_2(Y_1 + Y_{23})\right]}$$

Since $1/A_o << 1$ then

$$\frac{V_o}{V_1} \simeq \frac{Y_2 Y_4}{\left[Y_1(Y_2 + Y_3 + Y_4 + Y_{23}) + Y_2 Y_3\right] + \frac{1}{A_o}\left[Y_i(Y_2 + Y_3 + Y_4 + Y_{23}) + Y_2(Y_4 + Y_{23})\right]}$$

The assigned elements are substituted in the above equation and so

$$\frac{V_o}{V_1} \simeq \frac{C_2 G_4 s}{\left[C_2 C_3 s^2 + G_1(C_2+C_3)s + G_1(G_4+G_{23})\right] + \frac{1}{A_o}\left[G_i(C_2+C_3)s + C_2(G_4+G_{23})s + G_i(G_4+G_{23})\right]}$$

which may be written

$$\frac{V_o}{V_1} \simeq \frac{\frac{G_4}{C_3}s}{s^2 + \left[\left(G_1+\frac{G_i}{A_o}\right)\left(\frac{C_2+C_3}{C_2C_3}\right) + \frac{G_4+G_{23}}{A_oC_3}\right]s + \left[\left(G_1+\frac{G_i}{A_o}\right)(G_4+G_{23})\right]}$$

This is a second order bandpass filter. By specifying

$$2\sigma = \left[\left(G_1+\frac{G_i}{A_o}\right)\left(\frac{C_2+C_3}{C_2C_3}\right) + \frac{G_4+G_{23}}{A_oC_3}\right]$$

$$\omega_n^2 = \left[\left(G_1+\frac{G_i}{A_o}\right)(G_4+G_{23})\right]$$

Suitable values for C_2, C_3, G_1, G_4, G_{23} can be determined and the error due to the finite amplifier gain and finite input admittance reduced almost to zero. Because $G_i/A_o << G_1$, 2σ and ω_n^2 can be approximately written as

$$2\sigma \simeq \left[G_1\frac{C_2+C_3}{C_2C_3} + \frac{G_4+G_{23}}{A_oC_3}\right], \quad \omega_n^2 \simeq G_1(G_4+G_{23}) \tag{III.14}$$

$$Q \simeq \frac{\sqrt{G_1(G_4+G_{23})}}{\left[G_1\frac{C_2+C_3}{C_2C_3} + \frac{G_4+G_{23}}{A_oC_3}\right]} \tag{III.15}$$

If one defines $\Delta 2\sigma$ as the error of 2σ introduced by the finite gain of the amplifier, then from Eq. III.14

$$\Delta 2\sigma = \frac{G_4 + G_{2\cdot 3}}{A_o C_3} \qquad \text{(III.16)}$$

The sensitivity $S^{Q}_{A_O}$ is found from Eqs. III.14, III.15 and III.16

$$S^{Q}_{A_o} \simeq \frac{\Delta 2\sigma}{2\sigma} = \frac{1}{1 + \frac{A_o G_1 (C_2 + C_3)}{C_2 (G_4 + G_{2\cdot 3})}}$$

For a specific circuit both 2σ and Δ2σ is known, therefore Q sensitivity with respect to amplifier gain variations can be calculated.

c. Single-Ladder Synthesis

Single-ladder synthesis technique is due to Aggarwal [50,51,52,53,54,55], who has thoroughly investigated the technique with its special cases. The general circuit configuration of single-ladder synthesis is shown in Fig. III.10. For single-ladder synthesis the algorithm for

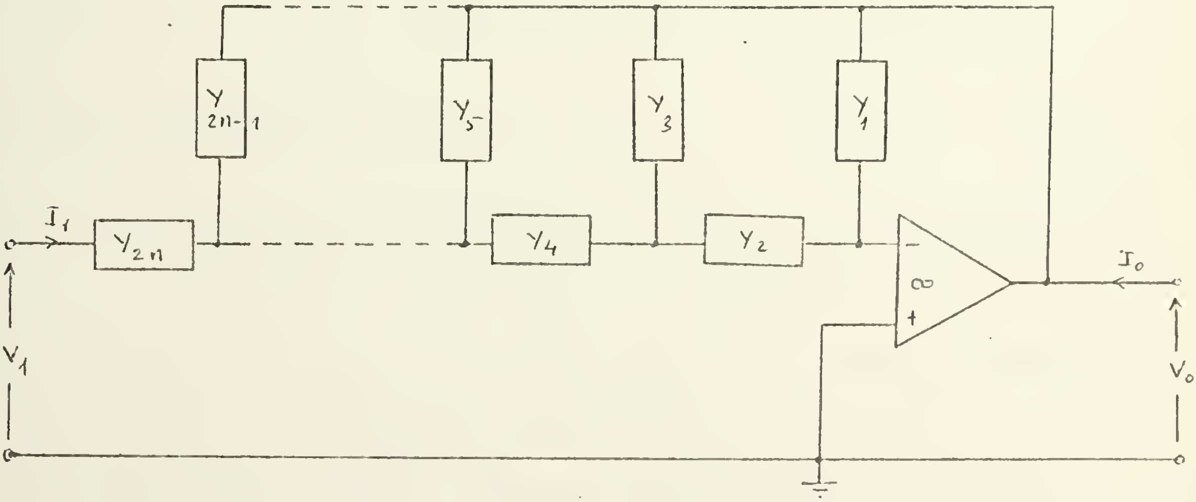


Figure III.10 Single-Ladder Configuration

finding the voltage transfer function is also due to Aggarwal [52].

As an example of this method, the same transfer function of Eq. III.12, which was used in the double-ladder method will be considered. Hence

$$T(s) = -\frac{b s^2}{a_3 s^3 + a_2 s^2 + a_1 s + 1}$$

The general circuit used for this synthesis is shown in Fig. III.11.

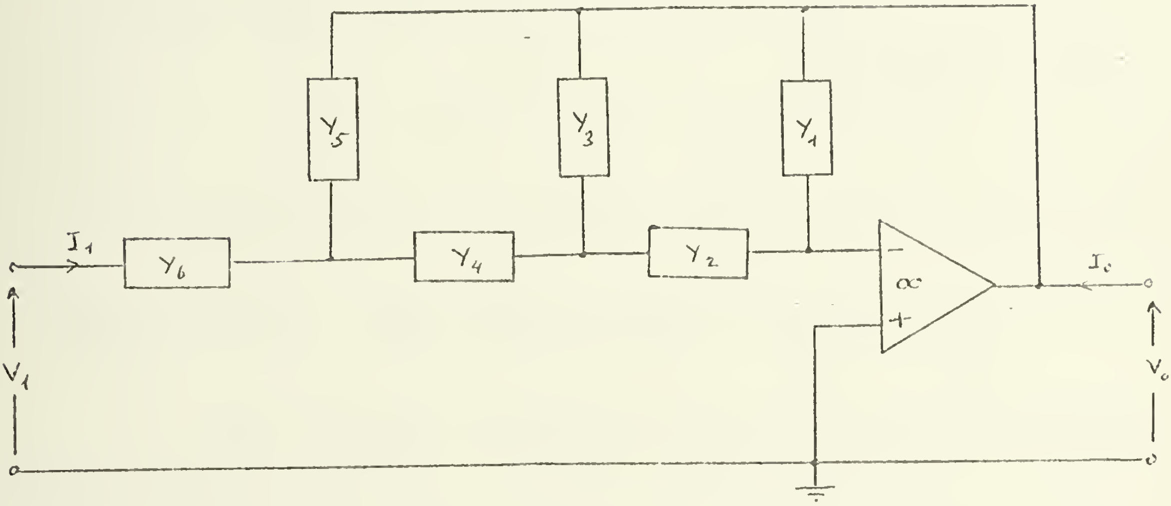


Figure III.11 A General Example of Single-Ladder Synthesis

The voltage transfer function is

$$\frac{V_o}{V_1} = -\frac{Y_2 Y_4 Y_6}{Y_1[(Y_2+Y_3+Y_4)(Y_5+Y_6)+Y_4(Y_2+Y_3)]+Y_2[Y_3(Y_4+Y_5+Y_6)+Y_4 Y_5]}$$

By selecting $Y_2 = C_2 s$, $Y_3 = C_3 s$, $Y_6 = C_6 s$ and the remaining conductances, the transfer function of the circuit has the same form as Eq. III.12. Equating coefficients yields

$$\alpha_1 = R_5(C_2+C_3+C_6) + R_4(C_2+C_3) + R_1C_2$$

$$a_2 = R_4R_5C_6(C_2+C_3) + R_1R_4C_2 + C_3 + R_1R_5C_2C_3$$

$$a_3 = R_1R_4R_5C_2C_3C_6 \quad , \qquad b = R_1R_5C_2C_6$$

Then the circuit becomes as shown in Figure III.12

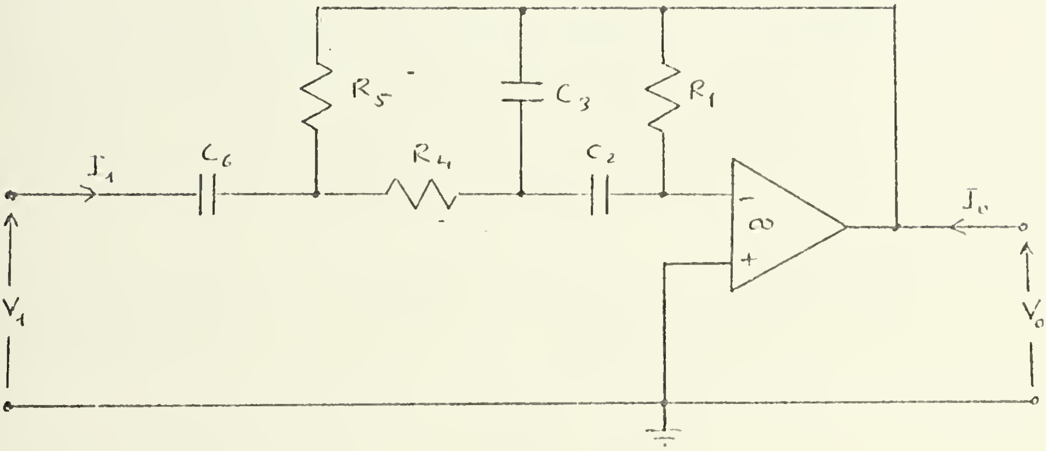


Figure III.12 The Final Circuit of the Example

For the same voltage transfer function a simpler circuit can be found and is shown in Fig. III.13

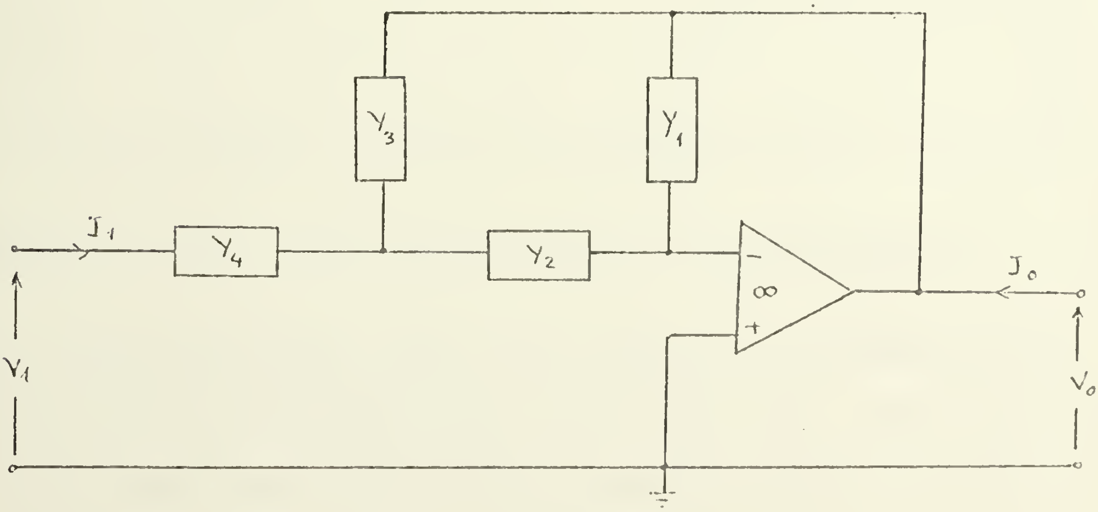


Figure III.13 A Second Example of Single-Ladder Synthesis.

The voltage transfer function is

$$\frac{V_o}{V_1} = -\frac{Y_2 Y_3}{Y_1 Y_2 + Y_1 Y_3 + Y_1 Y_4 + Y_2 Y_3}$$

The two terminal admittances are selected

$$Y_1 = G_1 + C_1 s , \qquad Y_2 = \frac{C_2 s}{R_2 C_2 s + 1}$$

$$Y_3 = G_3 \quad , \; Y_4 = C_4 s$$

Then the transfer function of the circuit satisfies the desired transfer function with the coefficients

$$a_1 = C_2 (R_1 + R_2 + R_3) + R_1 C_1 + R_3 C_4$$

$$a_2 = C_1 C_2 (R_1 R_2 + R_1 R_3) + R_1 R_3 C_1 C_4 + R_2 R_3 C_2 C_4$$

$$a_3 = R_1 R_2 R_3 C_1 C_2 C_4 \quad , \quad b = R_1 R_3 C_2 C_3$$

The final circuit is shown in Fig. III.14

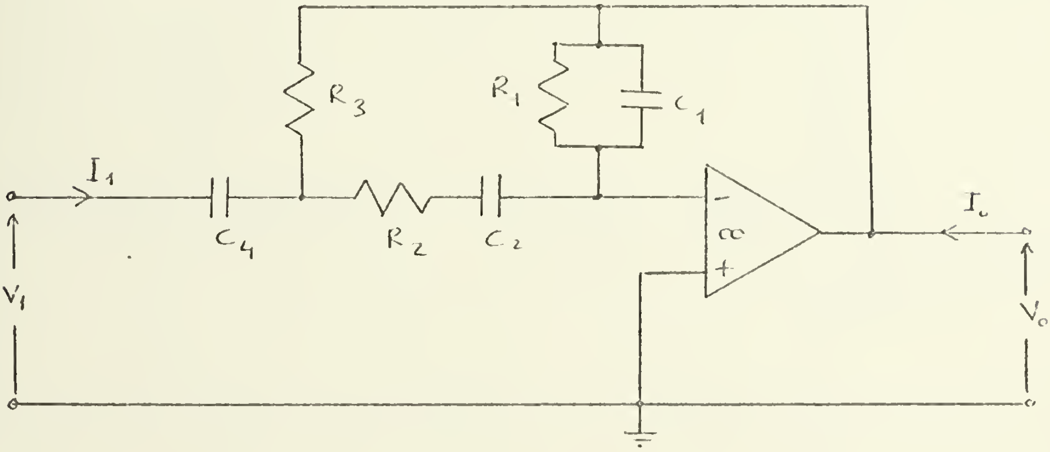


Figure III.14 The Final Circuit of Second Example

As is seen from Figs. III.12 and III.14, the second circuit has no obvious advantages over the first one. Both circuits use the same number of passive elements, but the second circuit was easier to analyze and the element values of one

circuit might be more convenient than those of the other.

A special case of the circuit of Fig. III.10, has been investigated by Aggarwal [55], because of its ease of analysis. This special circuit is shown in Fig. III.15 and may be used to synthesize n^{th} order voltage transfer functions

$$T(s) = -\frac{b_0(b_{n-1}s^{n-1} + b_{n-2}s^{n-2} + \cdots + b_1 s + 1)}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + 1}$$

where all of the numerator and denominator coefficients are real, non zero and positive. In this configuration it is assumed that the shunt elements,

$$Y_4 = Y_6 = Y_8 = \cdots = Y_{2n-2} = 0$$

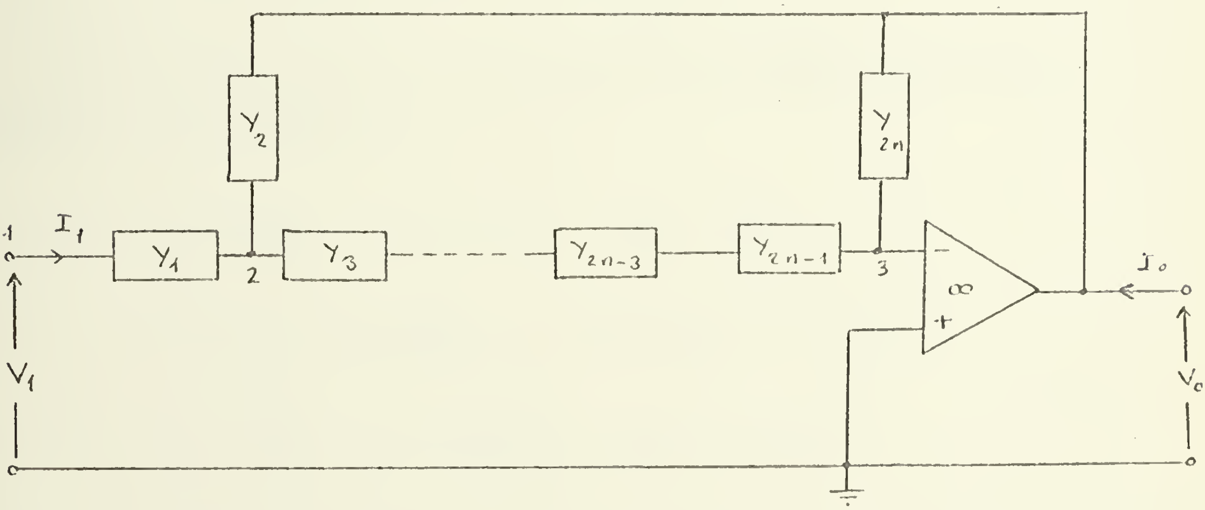


Figure III.15 A Special Case of Single-Ladder Synthesis

Then the admittance matrix of the circuit is (the procedure is described and proved in Appendix 1).

$$\begin{array}{c|ccc} & 1 & 2 & 4 \\ \hline 1 & Y_1 & -Y_1 & 0 \\ 2 & -Y_1 & Y_1+Y_2+\dfrac{1}{\sum\limits_{i=3}^{2n-1}\dfrac{1}{Y_i}} & -Y_2 \\ 3 & 0 & \dfrac{1}{\sum\limits_{i=3}^{2n-1}\dfrac{1}{Y_i}} & -Y_{2n} \end{array}$$

$i=3,5,7,\cdots,2n-1$

The voltage transfer function is

$$\frac{V_o}{V_1} = -\frac{Y_1}{\left(Y_2+Y_{2n}\right)+Y_{2n}\left(Y_1+Y_2\right)\sum\limits_{i=3}^{2n-1}\dfrac{1}{Y_i}}$$

Three combinations of this kind of realization which are easy to analyze are:

(1) $Y_1 = \frac{1}{R}$, $Y_2 = C_2 s$, $Y_{2n} = \frac{1}{q_{2n} R}$

$Y_i = C_i s + \frac{1}{q_i R}$ $\qquad i = 3,5,7,\cdots,2n-1$

(2) $Y_1 = \frac{1}{R}$, $Y_2 = \frac{1}{q_2 R}$, $Y_{2n} = C_{2n} s$

$Y_i = C_i s + \frac{1}{q_i R}$ $\qquad i = 3,5,7,\cdots,2n-1$

(3) $Y_1 = C_1 s + \frac{1}{q_1 R}$, $Y_2 = \frac{1}{q_2 R}$, $Y_3 = \frac{1}{R}$

$Y_{2n} = C_{2n} s$, $Y_i = C_i s + \frac{1}{q_i R}$ $\qquad i = 5,7,9,\cdots,2n-1$

As an illustration of the first combination a fourth order voltage transfer function is considered

$$T(s) = -\frac{b_o(b_3 s^3 + b_2 s^2 + b_1 s + 1)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + 1} \tag{III.17}$$

Since n=4, the voltage transfer function of the Fig. III.15 becomes

$$\frac{V_o}{V_1} = -\frac{Y_1}{(Y_2+Y_8) + Y_8(Y_1+Y_2)\sum_{i=3}^{7}\frac{1}{Y_i}}$$

Then by substituting the admittance values according to the first combination

$$\frac{V_o}{V_1} = -\frac{\frac{1}{R}}{\left(C_2 s + \frac{1}{q_8 R}\right) + \frac{1}{q_8 R}\left(\frac{1}{R} + C_2 s\right)\sum_{i=3}^{7}\frac{1}{C_i s + \frac{1}{q_i R}}} \tag{III.18}$$

After a few algebraic manipulations the coefficients of Eqs. III.17 and III.18 are compared yielding

$$d = \frac{1}{q_3 + q_5 + q_7 + q_8} \quad , \; b_o = q_8 d \quad , \; b_1 = R(q_3 C_3 + q_5 C_5 + q_7 C_7)$$

$$b_2 = R^2(q_3 q_5 C_3 C_5 + q_3 q_7 C_3 C_7 + q_5 q_7 C_5 C_7)$$

$$b_3 = R^3(q_3 q_5 q_7 C_3 C_5 C_7)$$

$$a_1 = d\Big\{q_8 b_1 + C_2 R(q_3 + q_5 + q_7 + q_8) + R\big[q_3(q_5 C_5 + q_7 C_7) +$$

$$q_5(q_3 C_3 + q_7 C_7) + q_7(q_3 C_3 + q_5 C_5)\big]\Big\}$$

$$a_2 = d\Big\{ q_8 b_2 + q_8^2 C_2 R b_1 + R^2 C_2 \big[q_3 (q_5 C_5 + q_7 C_7) + q_5 (q_3 C_3 + q_7 C_7) + $$

$$+ q_7 (q_3 C_3 + q_5 C_5) \big] + q_3 q_5 q_7 R^2 (C_3 C_5 + C_3 C_7 + C_5 C_7) \Big\}$$

$$a_3 = d\left[q_8 b_3 + q_8^2 C_2 R b_2 + R^3 C_2 q_3 q_5 q_7 (C_3 C_5 + C_3 C_7 + C_5 C_7) \right]$$

$$a_4 = b_0 q_8 b_3 C_2 R$$

For a desired transfer function, the coefficients of Eq. III.17 are known. A value for R is arbitrarily selected. Then there remain eight unknowns (q_3, q_5, q_7, q_8, C_2, C_3, C_5, C_7) and eight nonlinear algebraic equations which generally require a digital computer solution. After these algebraic equations are solved, the assignments for the element values are made. The resulitng circuit is shown in Fig. III.16.

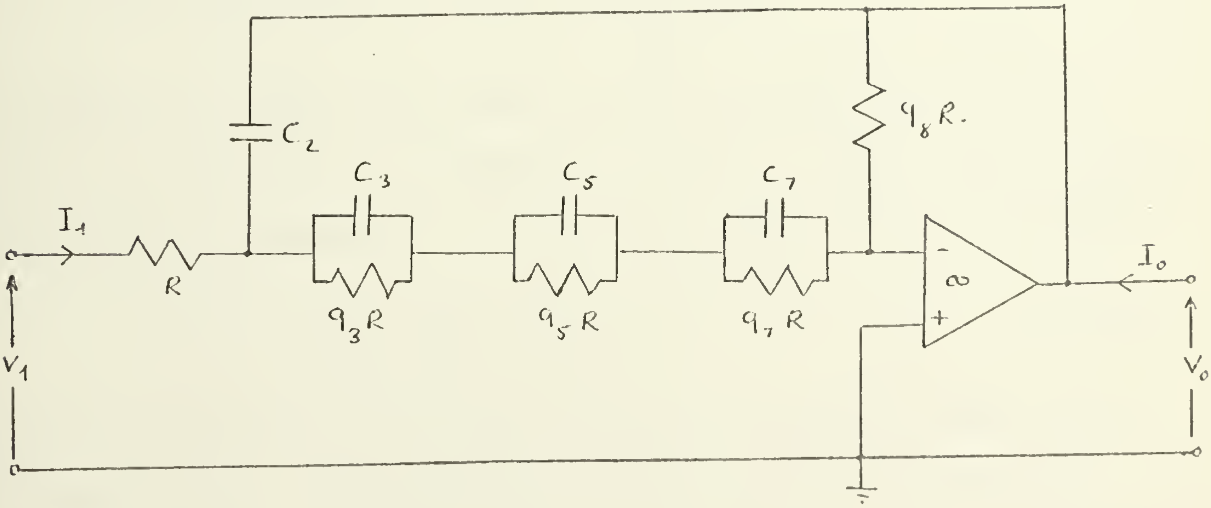


Figure III.16 An Example of Special Case

(1) Sensitivity Considerations. For the single-ladder synthesis it is also possible to calculate and eliminate the error due to the finite amplifier gain and finite input and output impedance.

For the special case shown in Fig. III.15 the voltage transfer function with finite gain and finite input admittance is found.

$$\frac{V_o}{V_1} = -\frac{Y_1}{\left(1-\frac{1}{A_o}\right)\left[(Y_2+Y_{2n})+Y_{2n}(Y_1+Y_2)\sum_{j=3}^{2n-1}\frac{1}{Y_j}\right]+\frac{1}{A_o}\left[Y_1+Y_i+Y_i(Y_1+Y_2)\sum_{j=3}^{2n-1}\frac{1}{Y_j}\right]}$$

Because $1/A_o << 1$, V_o/V_1 is approximated

$$\frac{V_o}{V_1} = -\frac{Y_1}{\left[(Y_2+Y_{2n})+Y_{2n}(Y_1+Y_2)\sum_{j=3}^{2n-1}\frac{1}{Y_j}\right]+\frac{1}{A_o}\left[Y_1+Y_i+Y_i(Y_1+Y_2)\sum_{j=3}^{2n-1}\frac{1}{Y_j}\right]} \qquad \text{(III.19)}$$

Thus the error due to finite gain and input impedance is

$$E = \frac{1}{A_o}\left[Y_1+Y_i+Y_i(Y_1+Y_2)\sum_{j=3}^{2n-1}\frac{1}{Y_j}\right]$$

For specific cases this error can be calculated and eliminated by adjusting the admittance values in the design process. For example consider the given second order transfer function

$$T(s) = \frac{Ks}{s^2+2\sigma s+\omega_n^2} \qquad \text{(III.20)}$$

Equation III.20 is to be realized by the special case shown in Fig. III.15. The admittances for n=2 are assigned as follows:

$$Y_1 = G \ , \ Y_2 = C_2 s \ , \ Y_3 = C_3 s \ , \ Y_4 = \frac{G}{q_4} \ .$$

The circuit with these elements is shown in Fig. III.17.

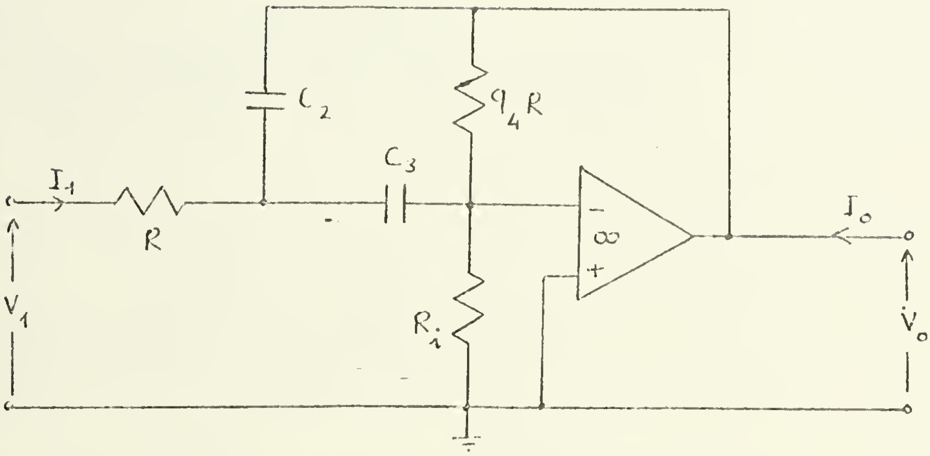


Figure III.17 A Second Order Single-Ladder System

The resistance R_i is the input impedance of the amplifier. By substituting for Y_j's and n into the Eq. III.19, the voltage transfer function of the network is found.

$$\frac{V_o}{V_1} \simeq - \frac{\frac{G}{C_2} s}{S^2 + \left[\frac{G_4}{q_4} \left(\frac{C_2 + C_3}{C_2 C_3} \right) + \frac{1}{A_o} \left(\frac{G}{C_2} + \frac{G_i}{C_3} \right) \right] s + \frac{G^2}{C_2 C_3} \left(\frac{1}{q_4} + \frac{G_i}{G A_o} \right)}$$

By assigning

$$2\sigma = \left[\frac{G}{q_4} \left(\frac{C_2 + C_3}{C_2 C_3} \right) + \frac{1}{A_o} \left(\frac{G}{C_2} + \frac{G_i}{C_3} \right) \right]$$

$$\omega_n^2 = \frac{G^2}{C_2 C_3} \left(\frac{1}{q_4} + \frac{G_i}{G A_o} \right) \simeq \frac{G^2}{C_2 C_3 q_4}$$

the error due to infinite gain and input impedance may be eliminated.

The Q is

$$Q \simeq \frac{\dfrac{G}{\sqrt{C_2 C_3 g_4}}}{\left[\dfrac{G}{g_4}\left(\dfrac{C_2 + C_3}{C_2 C_3}\right) + \dfrac{1}{A_0}\left(\dfrac{G}{C_2} + \dfrac{G_i}{C_3}\right)\right]}$$

$$S^{Q}_{A_0} \simeq \frac{\dfrac{1}{A_0}\left(\dfrac{G}{C_2} + \dfrac{G_i}{C_3}\right)}{2\sigma}$$

By defining $\Delta 2\sigma$ as the error of 2σ introduced by the finite gain of the amplifier

$\Delta 2\sigma = \dfrac{1}{A_0}\left(\dfrac{G}{C_2} + \dfrac{G_i}{C_3}\right)$. Then $S^{Q}_{A_0}$ can be

rewritten in the form

$$S^{Q}_{A_0} \simeq \frac{\Delta 2\sigma}{2\sigma} = \frac{1}{\left[1 + \dfrac{G A_0}{g_4}\left(\dfrac{C_2 + C_3}{G C_2 + G_i C_3}\right)\right]}$$

Note that this is the same expression obtained as in the double-ladder synthesis. However $\Delta 2\sigma$ is different for the two techniques

d. Brugler's and Bohn's Circuits

Two individual networks to realize voltage transfer functions were proposed by Bohn [43] and Brugler [21].

Bohn's circuit has been thoroughly investigated and eleven function realizations are catalogued by Holt and Sewell [56].

Bohn's circuit is shown in Fig. III.18. The transfer function of the circuit in Fig. III.18 is:

$$\frac{V_o}{V_1} = \frac{Y_3 Y_1 + Y_6 (Y_1 + Y_2 + Y_3 + Y_4)}{Y_3 Y_4 + Y_5 (Y_1 + Y_2 + Y_3 + Y_4)} \qquad \text{(III.21)}$$

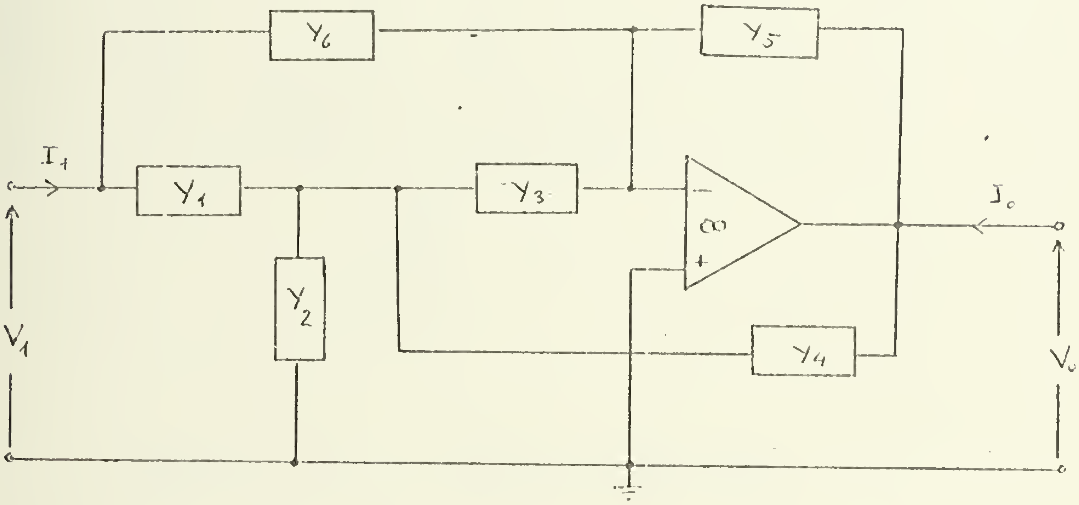


Figure III.18 Bohn's Circuit

Even though it is possible to realize functions of degree three and higher, Eq. III.21 is most suitable for second order systems. A given voltage transfer function $T(s) = N(s)/Q(s)$ is compared with Eq. III.21. By giving values to various admittances the coefficients of Eq. III.21 are matched with the coefficients of the desired transfer function and the synthesis is complete.

The second circuit was proposed by Brugler and shown in Fig. III.19. Note that both positive and negative feedback is employed, and that it is possible to realize any

kind of voltage transfer function, for example, there is no restriction on the location of poles and zeros.

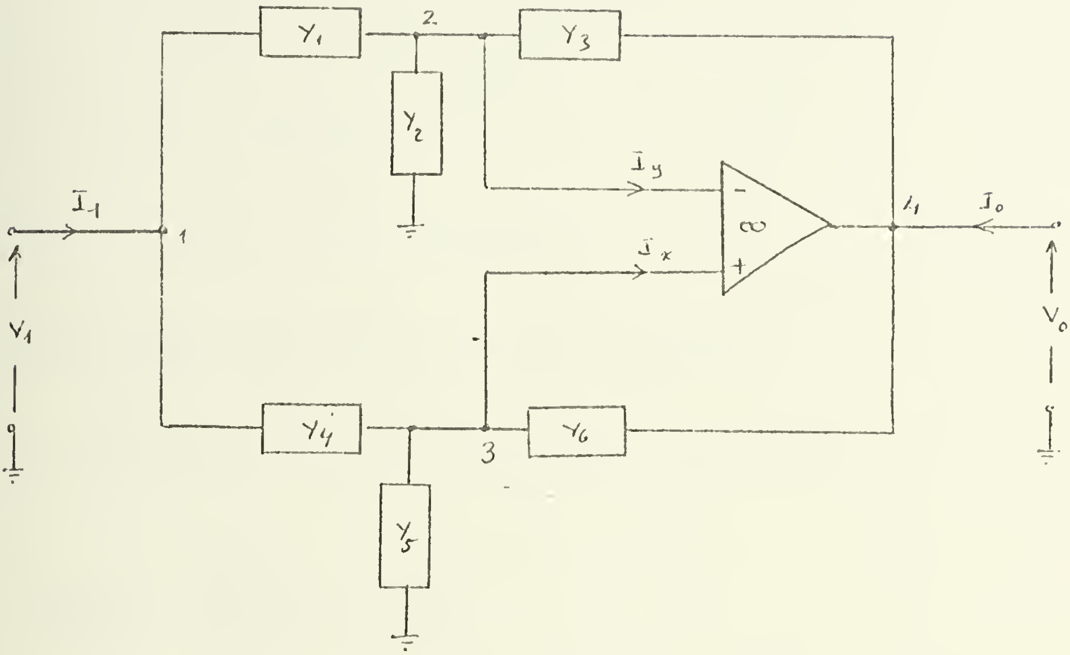


Figure III.19 Brugler's Circuit

From Fig. III.19 the nodal equations for nodes 2 and 3 are

$$(V_2 - V_1)Y_1 + V_2 Y_2 + (V_2 - V_4)Y_3 + I_y = 0$$

$$(V_3 - V_1)Y_4 + V_3 Y_5 + (V_3 - V_4)Y_6 + I_x = 0$$

For an ideal operational amplifier

$$I_x = 0, \quad I_y = 0, \quad V_2 - V_3 = 0$$

Manipulating the nodal equations the voltage transfer function of the circuit is obtained as

$$\frac{V_o}{V_1} = \frac{Y_1(Y_4 + Y_5 + Y_6) - Y_4(Y_1 + Y_2 + Y_3)}{Y_6(Y_1 + Y_2 + Y_3) - Y_3(Y_4 + Y_5 + Y_6)}$$

If the admittances are chosen such that

$$Y_1 + Y_2 + Y_3 = Y_4 + Y_5 + Y_6$$

then the transfer function becomes much simpler

$$\frac{V_0}{V_1} = \frac{Y_1 - Y_4}{Y_6 - Y_3} \tag{III.22}$$

Assuming a given transfer function

$$T(s) = \frac{N(s)}{Q(s)} = \frac{a_m s^m + \cdots + a_1 s + a_0}{b_n s^n + \cdots + b_1 s + b_0} \tag{III.23}$$

then a polynomial P(s) is selected such that

$$P(s) = \prod_{i=1}^{n-1} (s + c_i)$$

where n is the degree of Q(s), the c_i's are real, positive and distinct. Equations III.22 and III.23 are made equal to each other, yielding

$$\frac{Y_1 - Y_4}{Y_6 - Y_3} = \frac{N(s)}{Q(s)} = \frac{N(s)/P(s)}{Q(s)/P(s)}$$

The assignments for numerator and denominator are

$$Y_1 - Y_4 = N(s)/P(s) \quad , \quad Y_6 - Y_3 = Q(s)/P(s) \tag{III.24}$$

Equations III.24 may be expanded into partial fractions. The result of this partial fraction expansion gives both positive and negative terms. The positive terms are associated with Y_1 and Y_6, and the negative terms are associated with Y_3 and Y_4. Then the synthesis of the driving point admittances is carried out and the synthesis is complete.

(1) Sensitivity Considerations. The voltage transfer function of Bohn's circuit with a finite gain amplifier can be found as

$$T(s) = \frac{V_o}{V_1} = \frac{N(s)}{Q(s)\left[1 + \frac{1}{A_o}\left(1 + \frac{N(s)}{Q(s)} + Y_1 Y_2\right)\right]}$$

where N(s)/Q(s) is the voltage transfer function with infinite gain. Because the network topology is known at the beginning of the synthesis, it is again possible to eliminate the effect of nonideal amplifier. By defining

$$\Delta T(s) = \frac{1}{A_o}\left[1 + \frac{N(s)}{Q(s)} + Y_1 Y_2\right] Q(s)$$

the sensitivity of T(s) with respect to gain of the amplifier is

$$S_{A_o}^{T(s)} = \frac{\Delta T(s)}{T(s)} \simeq \frac{N(s)}{\left[1 + \frac{A_o}{\left(1 + \frac{N(s)}{Q(s)} + Y_1 Y_2\right)}\right]}$$

Both T(s) and ΔT(s) are known functions of s, once a particular transfer function is given, therefore $S_{A_o}^{T(s)}$ can be calculated.

The voltage transfer function of Brugler's network with nonideal amplifiers can be found and sensitivity functions can also be calculated. However since both numerator and denominator have difference terms, this technique inherently has high sensitivity with respect to amplifier gain. In the comparison of different active RC technique

it is stated that the best polynomial composition for the NIC's gives a Q sensitivity $S_k^Q > 2Q-1$ with respect to conversion factor k. Since Brugler's technique also uses polynomial decomposition it is possible to state that the sensitivity of Q of the pole pair for a given transfer function will be very high.

C. MULTIPLE OPERATIONAL AMPLIFIER SYNTHESIS

1. General

In this part active RC networks containing more than one operational amplifier are investigated. The transfer functions which can be realized with multiple operational amplifiers are usually more general than those which are realized with a single operational amplifier. Such transfer functions consist of the ratio of two polynomials with real coefficients. The coefficients of the numerator and the denominator polynomials must be real, however they can be positive, negative or zero. Therefore there is no constraint on the location of the poles or zeros and they might be anywhere in the s-plane. However for stable structures the poles of the transfer function are required to lie in the left half s-plane.

The multiple operational amplifier synthesis can be classified by two techniques. The first technique which was proposed by Kerwin, Huelsman and Newcomb [12] is somewhat parallel to the state variable technique in the sense that it uses summers and integrators only. The second technique due to Mathews and Seifert [57] and Lovering [58] uses some predetermined circuit configurations.

2. State Variable Synthesis

Consider the open circuit voltage transfer function

$$T(s) = \frac{V_o}{V_1} = \frac{a_0 + a_1 s + \cdots + a_n s^n}{b_0 + b_1 s + \cdots + b_n s^n}$$

where a_i and b_i, $i=1,2,\cdots,n$ are real but might be positive, negative or zero. Multiply both the numerator and denominator by some variable X which has yet to be defined. Then

$$\frac{V_o}{V_1} = \frac{a_0 X + a_1 s X + \cdots + a_{n-1} s^{n-1} X + a_n s^n X}{b_0 X + b_1 s X + \cdots + b_{n-1} s^{n-1} X + b_n s^n X} \qquad \text{(III.25)}$$

The Eq. III.25 is separated into two parts

$$V_o = a_0 X + a_1 s X + \cdots + a_{n-1} s^{n-1} X + a_n s^n X \qquad \text{(III.26a)}$$

$$V_1 = b_0 X + b_1 s X + \cdots + b_{n-1} s^{n-1} X + b_n s^n X \qquad \text{(III.26b)}$$

The Eq. III.26b is rewritten

$$s^n X = -\frac{b_0}{b_n} X - \frac{b_1}{b_n} s X - \cdots - \frac{b_{n-1}}{b_n} s^{n-1} X + \frac{1}{b_n} V_1 \qquad \text{(III.27)}$$

This equation resembles a state equation in s if the states are defined as follows

$$Y_1 = X$$

$$Y_2 = sY_1 = sX$$

$$Y_3 = sY_2 = s^2 X$$

$$- - - - - - - - - -$$

$$Y_n = sY_{n-1} = s^{n-1} X$$

Then Eq. III.27 can be written in matrix form as a state equation.

$$
s\underset{\sim}{Y} = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdot & 0 \\ 0 & 0 & 1 & 0 & \cdot & 0 \\ 0 & 0 & 0 & 1 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot & 1 \\ -\frac{b_0}{b_n} & -\frac{b_1}{b_n} & -\frac{b_2}{b_n} & \cdot & \cdot & -\frac{b_{n-1}}{b_n} \end{bmatrix} \underset{\sim}{Y} + \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ \frac{1}{b_n} \end{bmatrix} V_1 \qquad \text{(III.28)}
$$

Also, Eq. III.26a can be rewritten as

$$
V_o = a_0 Y_1 + a_1 Y_2 + \cdots\cdots + a_{n-1} Y_n + a_n s Y_n \qquad \text{(III.29)}
$$

which resembles the output of the system described by the Eq. III.28. The signal flow graph for the system defined by Eq. III.28 and III.29 is shown in Fig. III.20, and the circuit realization of this signal flow graph is shown in Fig. III.21.

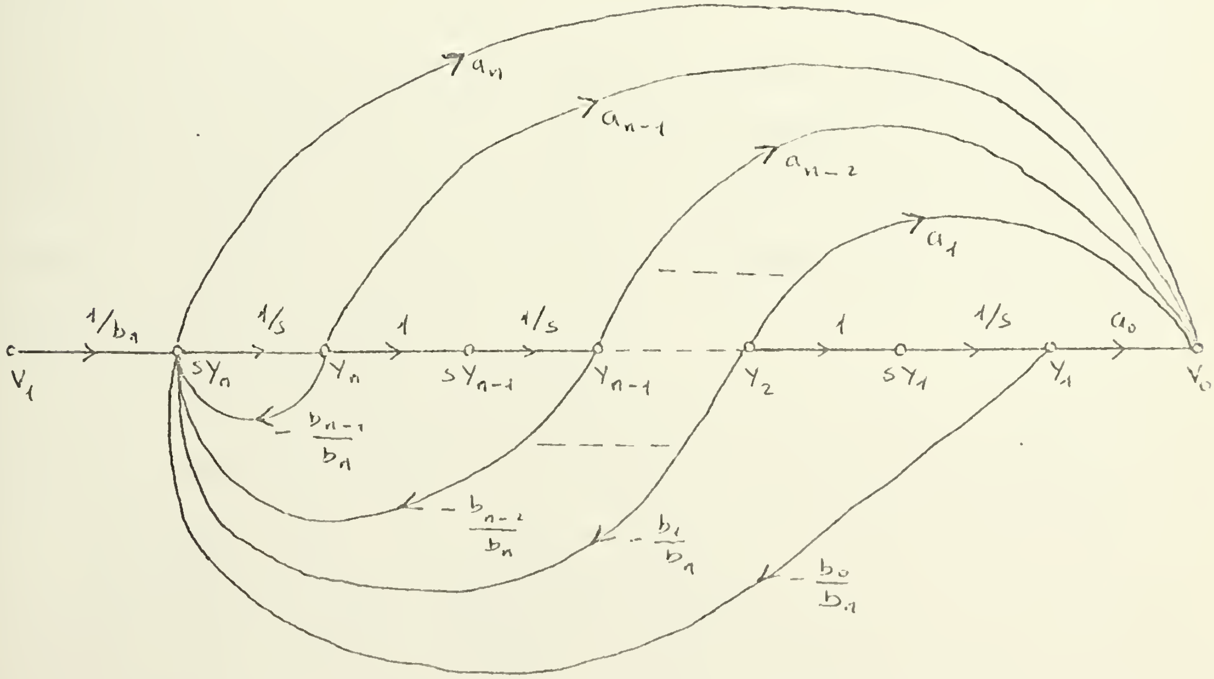


Figure III.20 The Signal Flow Graph of the n^{th} Degree System

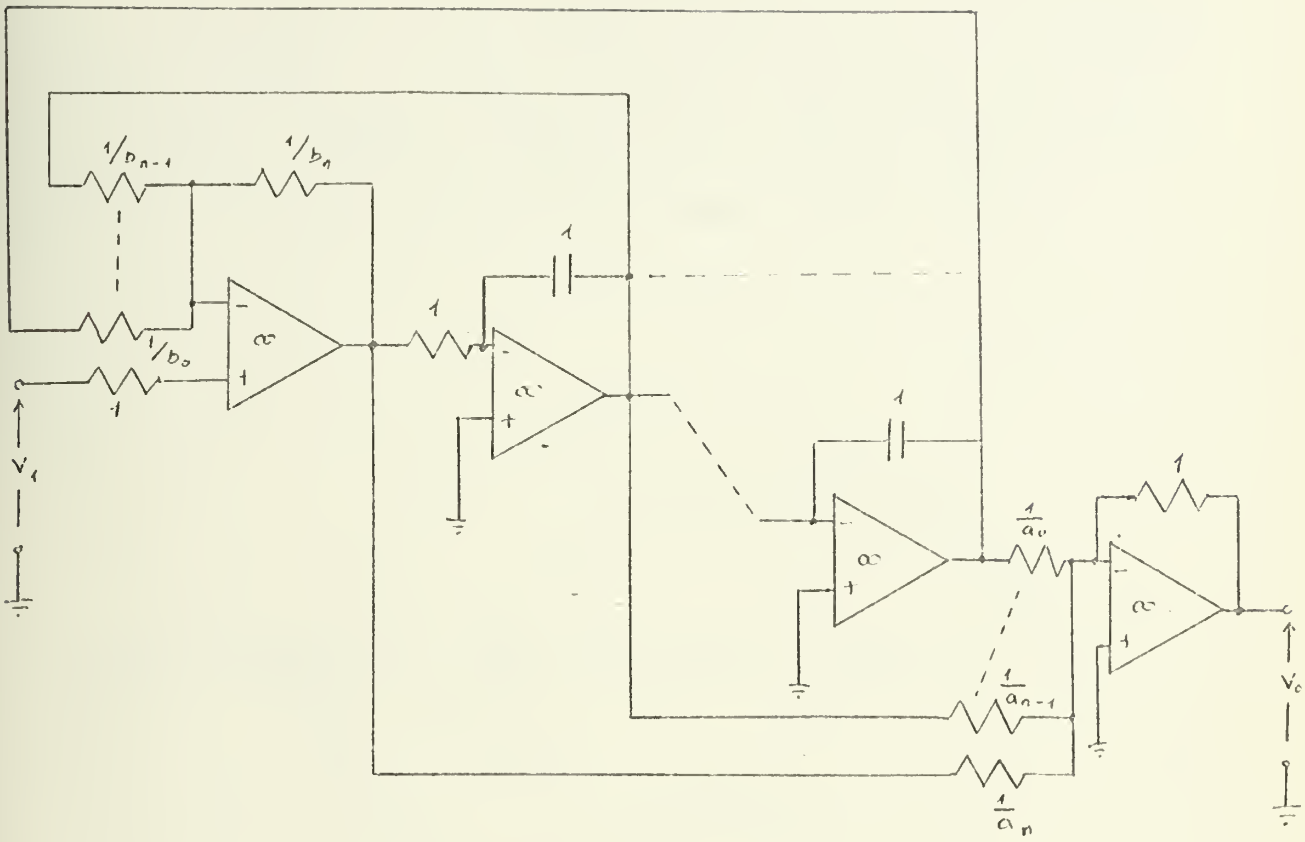


Figure III.21 The Circuit Realization of the Signal Flow Graph of Fig. III.20

The second degree voltage transfer function realization by this technique is of interest because of the simultaneous availability of the lowpass, bandpass and highpass functions from the single circuit. Assume that a second degree voltage transfer is given,

$$T(s) = \frac{V_o}{V_1} = \frac{a_0 + a_1 s + a_2 s^2}{b_0 + b_1 s + b_2 s^2}$$

Then the state and the output equations for this system are

$$\begin{bmatrix} sY_1 \\ sY_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{b_0}{b_2} & -\frac{b_1}{b_2} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{b_2} \end{bmatrix} V_1$$

$$V_o = a_0 Y_1 + a_1 Y_2 + a_2 s Y_2$$

The signal flow graph for this case is shown in Fig. III.22. Without knowing the signs of the coefficients of the desired transfer function it is not possible to draw the exact circuit diagram. However assuming all of the coefficients are positive, the circuit diagram is shown in Fig. III.23.

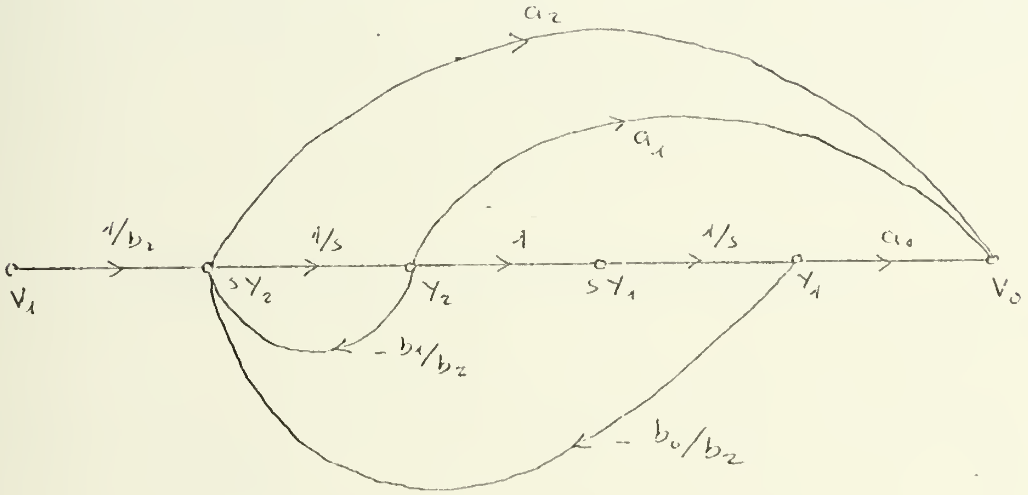


Figure III.22 The Signal Flow Graph for the Second Order System.

If a lowpass and/or a bandpass and/or a highpass function but not a combination of them (such as an all-pass) is required, then the amplifier A_4 is not necessary and can be removed. Then the lowpass, bandpass and the highpass transfer functions can be obtained simultaneously as shown in Fig. III.24. In both Fig. III.23 and III.24 the relations between resistors R_1 through R_4 are

$$\frac{R_4}{R_1} = H \cdot \frac{b_0}{b_2} \quad , \quad \frac{R_4}{R_2} = H \cdot \frac{1}{b_2} \quad , \quad \frac{R_4 + R_3}{R_3} = H \frac{b_1}{b_2}$$

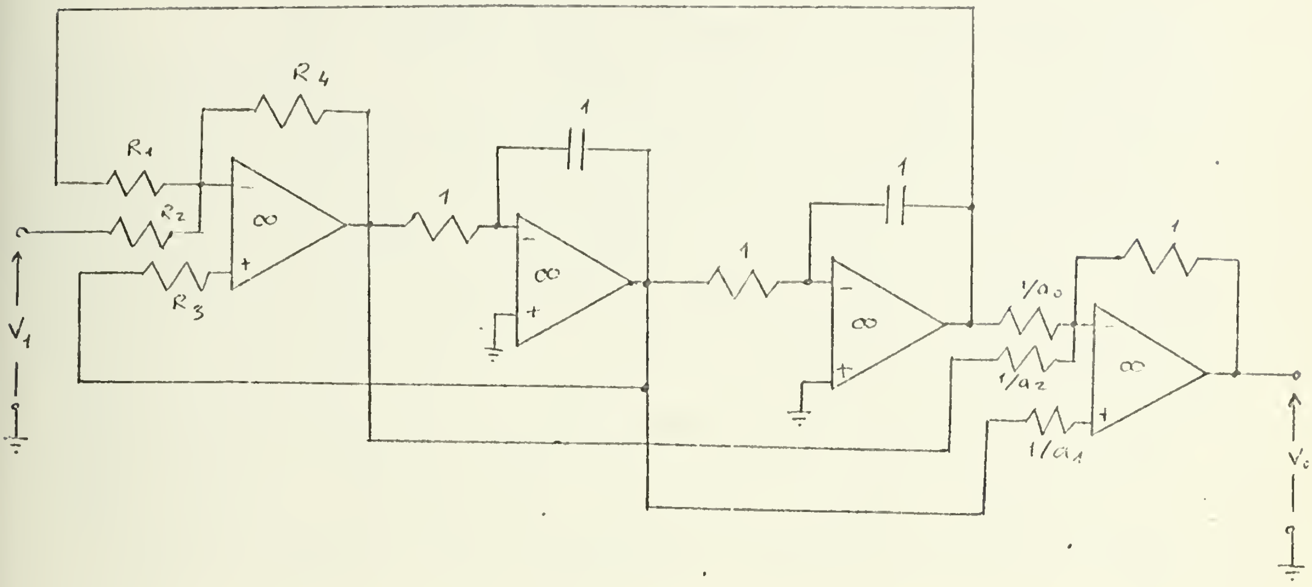


Figure III.23 A Second Order System

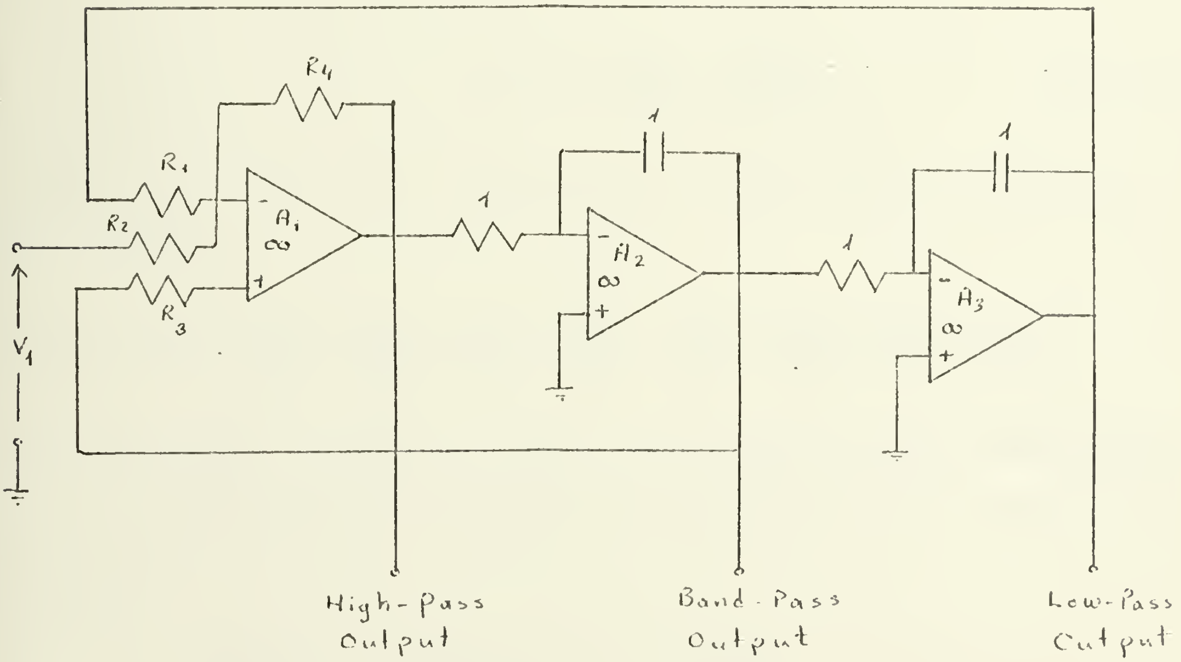


Figure III.24 A Simultaneous H-P, B-P, L-P, Second Order Filter.

where H is a positive multiplication factor to ensure that $Hb_1/b_2 \geqslant 1$.

a. Sensitivity Considerations

For a second order system realization consider the amplifiers A_1, A_2, A_3 in Fig. III.24 to have finite gain. Because the gain of the first amplifier A_1 only affects the multiplication factor H any gain variations in A_1 will not change pole or zero locations or the Q of the circuit. However the gain variations of the two integrator amplifiers A_2 and A_3 is of interest, because they effect pole-zero locations and Q. The denominator of the transfer function of III.28 with finite gain integrator amplifiers may be shown to be

$$D(s) = \left(b_2 + \frac{b_2}{A_2} + \frac{b_2}{A_3} + \frac{b_2}{A_2A_3}\right)s^2 + \left(b_1 + \frac{b_1}{A_3} + \frac{b_2}{A_2} + \frac{b_2}{A_3} + \frac{2b_2}{A_2A_3}\right)s + \left(b_0 + \frac{b_1}{A_3} + \frac{b_2}{A_2A_3}\right) \qquad \text{(III.30)}$$

Neglecting terms with denominators containing the product A_2A_3 gives

$$D(s) \simeq \left(b^2 + \frac{b_2}{A_2} + \frac{b_2}{A_3}\right)s^2 + \left(b_1 + \frac{b_1}{A_3} + \frac{b_2}{A_2} + \frac{b_2}{A_3}\right)s + \left(b_0 + \frac{b_1}{A_3}\right)$$

The relations between the root sensitivities and coefficients of a polynomial were mentioned in Section II.C.7 and are here repeated

$$\sum_{j=1}^{n} S_k^{P_j} = \frac{k(c_n b_{n-1} - c_{n-1} b_n)}{(b_n + k c_n)^2} ,$$

where k is any circuit parameter. The coefficient of the n^{th} term can be expressed as $a_n = (b_n + kc_n)$. Thus the pole

sensitivity of Eq. II.30 with respect to A_2 and A_3 is found to be

$$\sum_{i=1}^{2} S^{P_i}_{1/A_2} = \frac{\frac{1}{A_1}\left[b_1 b_2 + \frac{b_1 b_2}{A_3} - b_2^2\right]}{\left[b_2 + \frac{b_2}{A_2} + \frac{b_2}{A_3}\right]^2} \simeq \frac{1}{A_2}\left(\frac{b_1}{b_2} - 1\right)$$

$$\sum_{i=1}^{2} S^{P_i}_{1/A_3} = \frac{-\frac{1}{A_3}\left[b_2^2 - \frac{b_1 b_2}{A_2}\right]^2}{\left[b_2 + \frac{b_2}{A_2} + \frac{b_2}{A_3}\right]^2} \simeq -\frac{1}{A_3}$$

Because $S^{P_i}_{1/k} = -S^{P_i}_{k}$

$$\sum_{i=1}^{2} S^{P_i}_{A_2} \simeq \frac{1}{A_2}\left(1 - \frac{b_1}{b_2}\right) \quad , \quad \sum_{i=1}^{2} S^{P_i}_{A_3} \simeq \frac{1}{A_3}$$

Thus the circuit shown in Fig. III.24 has low sensitivities with respect to gain variations. As may be seen from Eq. III.30 the error introduced by assuming infinite gain is also extremely small for amplifiers with a gain of 10^3 or higher.

3. Mathews-Seifert's and Lovering's Circuit

In the single amplifier single feedback synthesis discussed in Section III.B.2, it was shown that if the poles and zeros of a given transfer function are complex, then Guillemin's or Fialkow-Gerst technique must be resorted to. These two techniques are lengthy and the final configuration is not known at the start. Therefore there is no way one can

estimate and correct for the effect of the finite amplifier gain.

Mathews and Seifert [57] proposed a circuit to replace the passive networks of the single feedback technique by simple zero producing active networks. Such a circuit is shown in Fig. II.25.

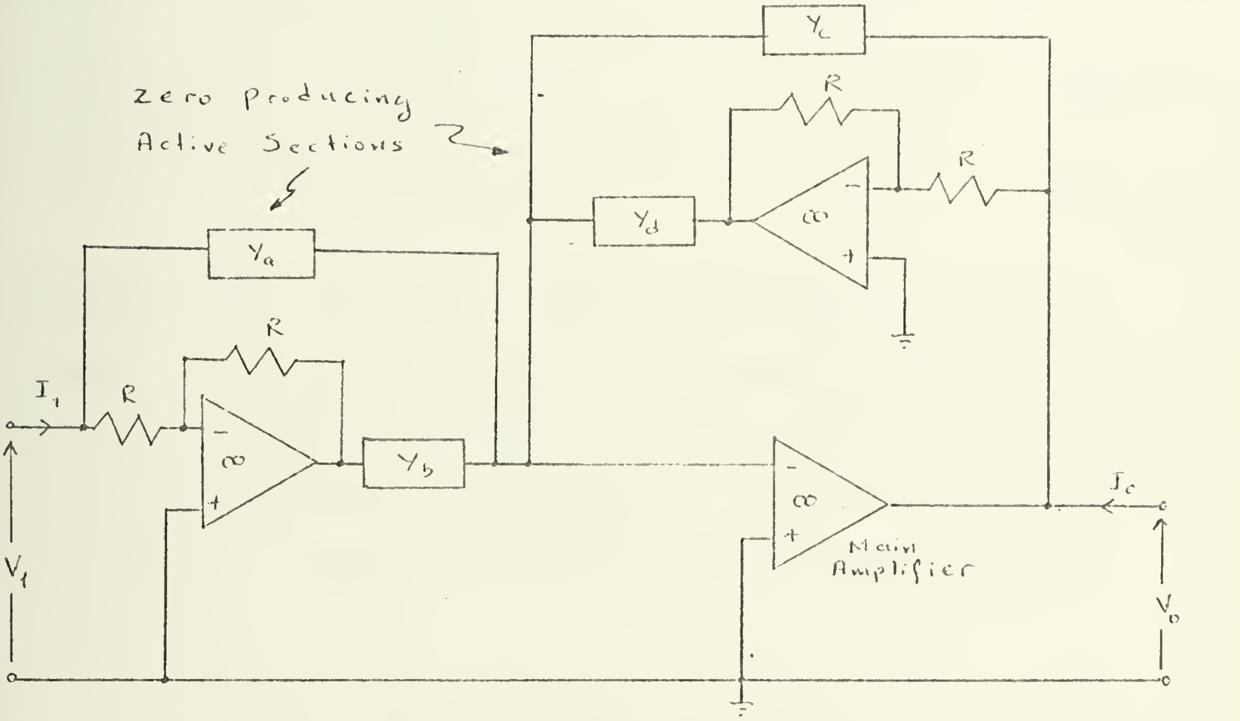


Figure III.25 Mathews-Seifert's Circuit

The voltage transfer function for this circuit is

$$\frac{V_o}{V_1} = \frac{Y_b - Y_a}{Y_c - Y_d}$$

Assume that a rational polynomial T(s) = N(s)/D(s) with real coefficients is desired to be realized as a voltage transfer function. A polynomial P(s) has then to be selected such that

$$P(s) = \prod_{r=1}^{n-1} (s + a_r)$$

where n is the degree of D(s), coefficients a_r are real positive, and distinct, but otherwise arbitrary. Dividing the numerator and the denominator of the transfer function by P(s) yields

$$\frac{Y_b - Y_a}{Y_c - Y_d} = \frac{N(s)/P(s)}{D(s)/P(s)}$$

By assigning the admittances

$$Y_b - Y_a = N(s)/P(s) \qquad \text{(III.31)}$$

$$Y_c - Y_d = D(s)/P(s) \qquad \text{(III.32)}$$

Expanding Eqs. III.31 and III.32 into partial fractions, positive and negative parts of the equations are obtained. The positive parts are associated with Y_b and Y_c, and negative parts are associated with Y_a and Y_d. Then the synthesis of driving point admittances can be made by Foster or Cauer forms.

Although Mathews-Seifert's circuit eliminates the difficulty of passive three terminal RC network synthesis, the circuit uses three active devices. Lovering [58] proposed a simpler circuit shown in Fig. III.26, to reduce the number of active elements. The voltage transfer function of Lovering's circuit is

$$\frac{V_o}{V_1} = \frac{Y_1 Y_5 - Y_2 Y_3}{Y_3 Y_6 - Y_4 Y_5}$$

By selecting Y_3 and Y_5 equal to each other, but otherwise arbitrary, the voltage transfer function becomes

$$\frac{V_o}{V_1} = \frac{Y_1 - Y_2}{Y_6 - Y_4}$$

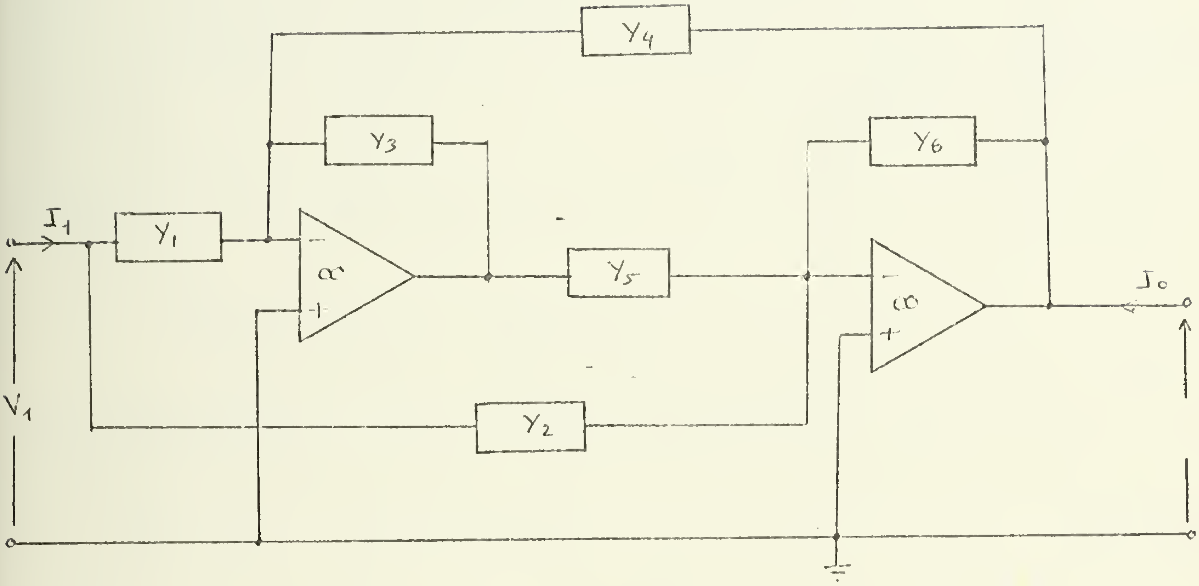


Figure III.26   Lovering's Circuit

Then the driving point admittances can be synthesized as in Mathews-Seifert's circuit.

A modification to Lovering's circuit can be made as shown in Fig. III.27. The voltage transfer function of this circuit is found to be

$$\frac{V_o}{V_1} = \frac{Y_b - Y_a}{Y_c - Y_b}$$

The transfer function which is to be realized is $T(s) = N(s)/D(s)$. The polynomial $P(s)$ is selected as specified in Mathews-Seifert's circuit. Then three new driving point admittances Y_a', Y_b' and Y_c' are defined such that

$$Y_b' - Y_a' = Y_b - Y_a \quad , \quad Y_c' - Y_b' = Y_c - Y_b$$

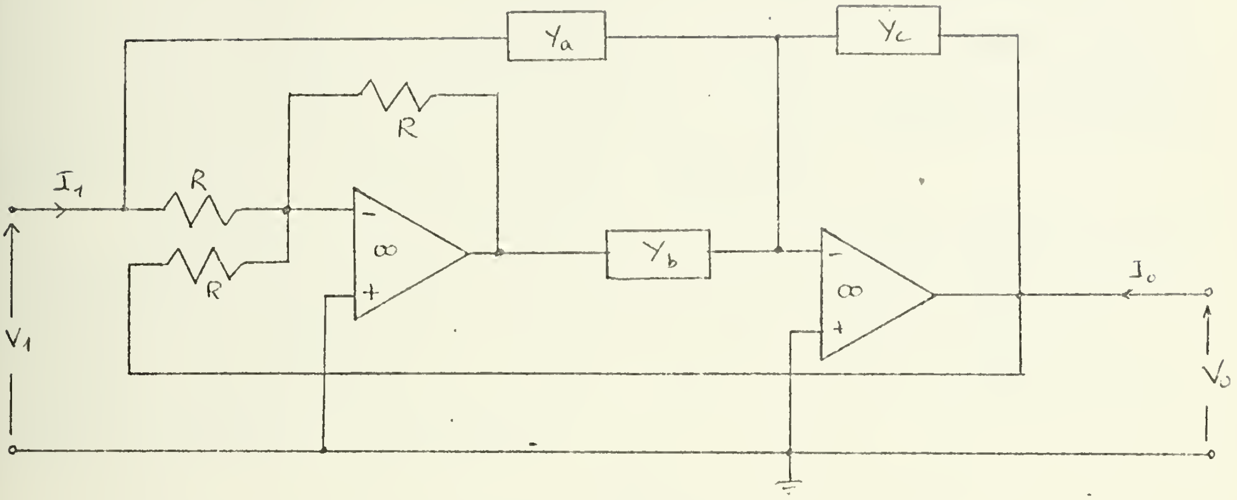


Figure III.27 Modified Lovering's Circuit

The assignments made for these admittances are

$$Y_b' - Y_a' = N(s)/P(s) \tag{III.33}$$

$$Y_c' - Y_b' = D(s)/P(s) \tag{III.34}$$

The Eqs. III.33 and III.34 are expanded into partial fractions as before. The sum of the positive parts are defined as $[N(s)/P(s)]^+$, and $[D(s)/P(s)]^+$. The sum of the negative parts are defined as $[N(s)/P(s)]^-$, and $[D(s)/P(s)]^-$. Then the Eqs. III.33 and III.34 are rewritten

$$Y_b' - Y_a' = [N(s)/P(s)]^+ + [N(s)/P(s)]^-$$

$$Y_c' - Y_b' = [D(s)/P(s)]^+ + [D(s)/P(s)]^-$$

and Y_a', Y_b', Y_c' are found as follows

$$Y_a' = [D(s)/P(s)]^- + [N(s)/P(s)]^-$$

$$Y_b' = [D(s)/P(s)]^- + [N(s)/P(s)]^+$$

$$Y_c' = [D(s)/P(s)]^+ + [N(s)/P(s)]^+$$

After Y_a', Y_b', Y_c' have been found, the driving point admittances Y_a, Y_b, Y_c can be found by removing the common terms from Y_a', Y_b' and Y_c' and the synthesis of Y_a, Y_b and Y_c can be done in terms of Foster or Cauer sections.

Because of the difference terms of both numerator and denominator polynomials, the last three circuits have inherently high sensitivities with respect to both active and passive element variations.

D. REALIZATION OF CONTROLLED SOURCES

Even though controlled source synthesis is not directly related to the active RC network synthesis with operational amplifiers, controlled sources built by operational amplifiers prove to have the least sensitivity. Hence controlled sources can be realized by means of operational amplifiers. Controlled source realizations with operational amplifiers will now be briefly discussed.

A voltage controlled voltage source (VCVS) is just a simple feedback operational amplifier as shown in Fig. III.28.

It should be noted that the gain of the positive gain VCVS is always greater than one. Furthermore, the gains are ratios of resistance, which if implemented by integrated circuits can be kept to within one percent tolerances.

A voltage controlled current source (VCCS) is shown in Fig. III.29, and a current controlled voltage source (CCVS) is shown in Fig. III.30.

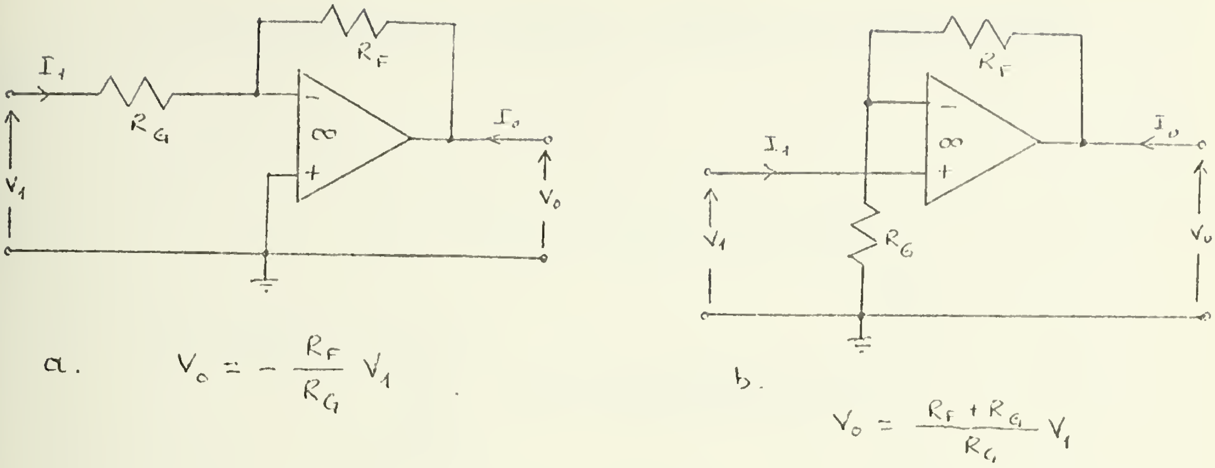


Figure III.28 VCVS Realizations: (a) Negative Gain (b) Positive Gain

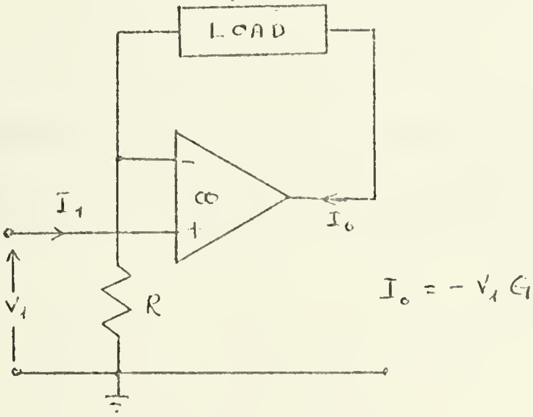


Figure III.29 A VCCS Realization

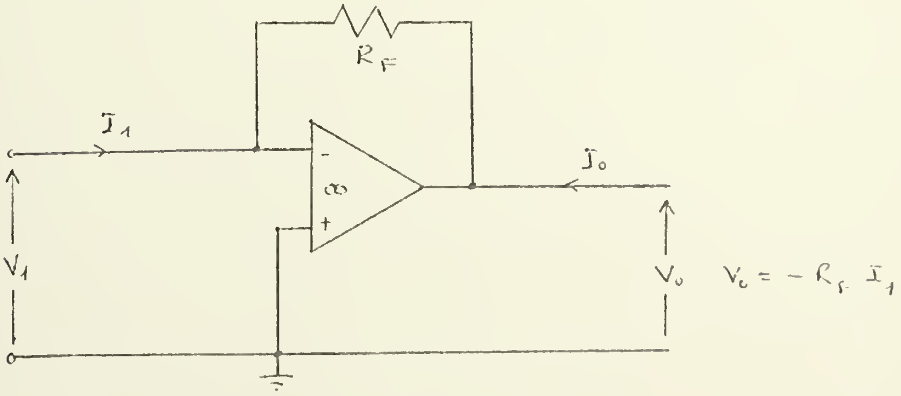


Figure III.30 A CCVS Realization

A current controlled current source is shown in Fig. III.31.

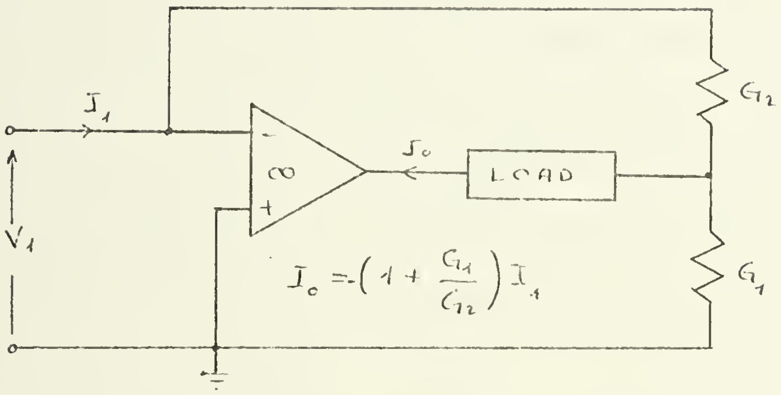


Figure III.31 A CCCS Realization

Note that the transfer function is again utilizing the ratio of two conductances and so is particularly suitable for integration.

IV. DESIGN EXAMPLES AND EXPERIMENTAL RESULTS

A. INTRODUCTION

This part of the thesis presents experimental results on circuits synthesized with R, C and operational amplifiers, according to the theory given in Section III. Six circuits are realized and tested, and experimental results are presented both in the form of a table and as a graph. Theoretical calculations are also given and plotted on the same graph as the experimental results.

The apparatus for measuring the circuits is shown in Fig. IV.1. The measuring arrangement consists, apart from two DC ± 15 volt power supplies, of an input signal generator, a vacuum tube voltmeter to measure the constant input voltage V_i and a cathode ray oscilloscope to measure the output voltage V_o and to check that no distortion occurred in the output due to overload, erroneous biasing or other causes. In all of the experimental circuits except circuit No. 3, the Donner Model 1202 sine wave generator was used. For circuit No. 3 the Hewlett-Packard Model 202A sine wave generator was used, because its frequency has a lower range than the Donner instrument. The output impedance of the Donner 1202 sine wave generator was measured to be about 500 ohms between 10 Hz to 1 MHz. In order to simulate an ideal voltage generator of internal resistance of 500 ohms the input voltage to the active RC circuit was kept constant by adjusting the output of the generator.

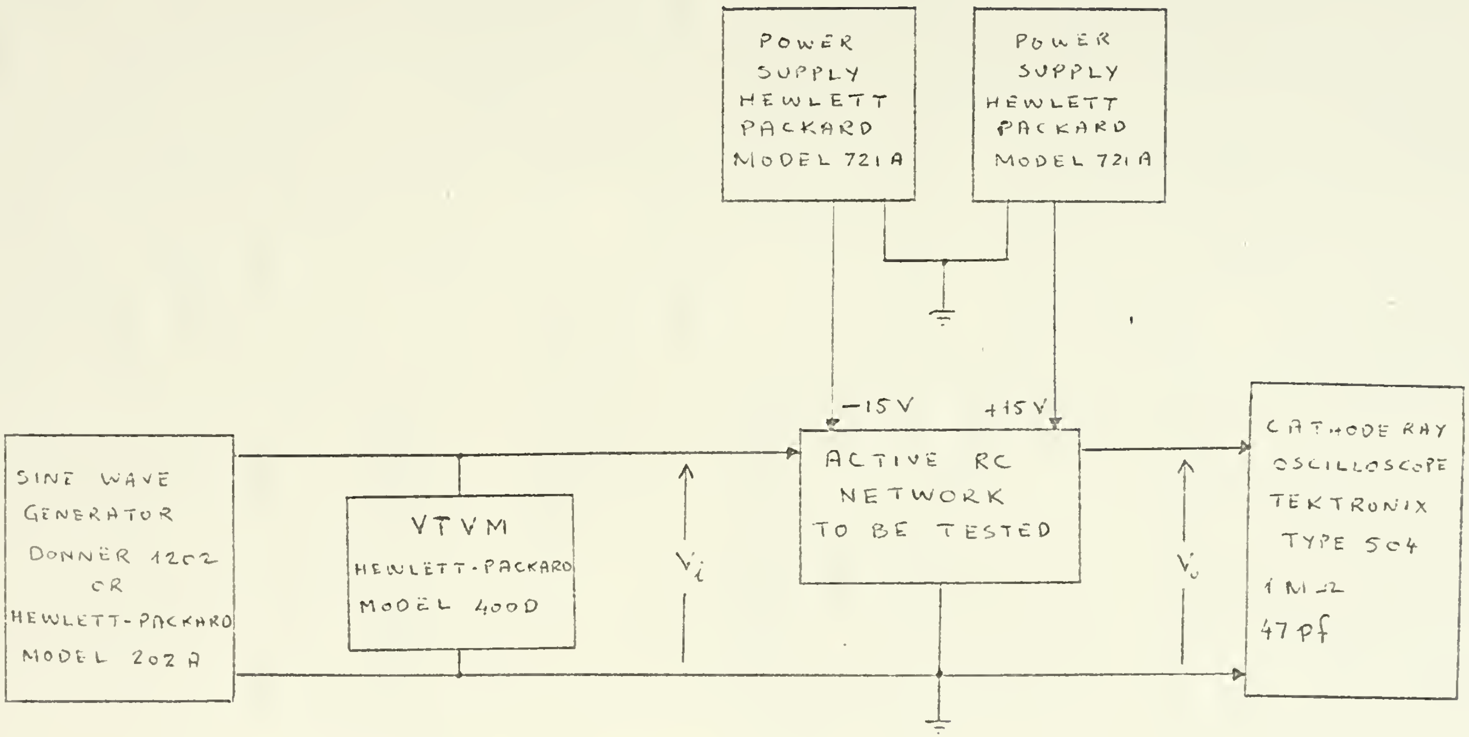

POWER SUPPLY HEWLETT PACKARD MODEL 721 A
POWER SUPPLY HEWLETT PACKARD MODEL 721 A
−15 V
+15 V
SINE WAVE GENERATOR DONNER 1202 OR HEWLETT-PACKARD MODEL 202 A
VTVM HEWLETT-PACKARD MODEL 400D
V_i
ACTIVE RC NETWORK TO BE TESTED
V_o
CATHODE RAY OSCILLOSCOPE TEKTRONIX TYPE 504
1 MΩ
47 pf

Figure IV.1 The Apparatus for Testing the Realized Circuits

In all the circuits tested the output terminal was also always made the output terminal of one of the operational amplifiers. Because the output impedance of the operational amplifier is typically 75 ohms, the oscilloscope with its high input impedance had no appreciable loading effect on the circuit, and so the error due to finite input impedance of the scope may be neglected. For each measurement taken the frequency of the signal generator is also measured with the scope to ensure frequency accuracy.

The accuracy of the measurement gear was as follows:

Frequency accuracy 1%	Tektronix type 504 CRO
Input Voltage accuracy ± 2%	Hewlett Packard Model 400D VTVM
Output Voltage accuracy 3%	Tektronix type 504 CRO
Bias Voltage Tolerances ± 0.3%	Hewlett Packard Model 721A Power Supplies

Passive Component Tolerances:

Resistors ± 5%	Allen-Bradley
Capacitors ± 5%	Cornell-Dubilier

Active Components:

Operational Amplifier - Fairchild μA741C data sheet given in Appendix II.

Because of the wide tolerances of the components with which the active RC networks are realized, there is some discrepancy between experimental results and the theoretical responses. However the results showed that the circuits realized with single amplifier, single feedback technique or double or single ladder techniques are less sensitive to element variations than the Brugler's or Lovering's structures.

B. CIRCUITS

1. Example No. 1: Single Feedback Synthesis

The open circuit voltage transfer function T(s) is to be realized by the single operational single feedback technique described in Section III.B.2. The function is second order bandpass and given by

$$T(s) = \frac{N(s)}{D(s)} = -\frac{1.15 \times 10^{4} s}{s^{2} + 1.26 \times 10^{4} s + 1.58 \times 10^{10}}$$

This transfer function has a bandpass characteristic with center frequency of twenty kilohertz at which the operational amplifier gain already departs from its high value and a quality factor of ten. The passive three terminal RC networks are to be realized by the Fialkow-Gerst technique described in Section III.A.6b.

An auxiliary denominator polynomial M(s) of the transfer function is selected.

$$M(s) = s^{2} + 3 \times 10^{5} s + 2 \times 10^{10} = (s + 1 \times 10^{5})(s + 2 \times 10^{5})$$

Then the open circuit voltage transfer functions of the input, and the feedback circuits shown in Fig. IV.4 become respectively

$$T_{12b}(s) = \frac{N(s)}{M(s)} = -\frac{1.15 \times 10^{4} s}{s^{2} + 3 \times 10^{5} s + 2 \times 10^{10}}$$

$$T_{12a}(s) = \frac{D(s)}{M(s)} = -\frac{y_{12a}}{y_{22a}} = -\frac{s^{2} + 1.26 \times 10^{4} s + 1.58 \times 10^{10}}{s^{2} + 3 \times 10^{5} s + 2 \times 10^{10}}$$

In order to define the short circuit input admittance of the feedback network it is necessary to select another polynomial

P(s) as described before. For this example P(s) is selected as follows

$$P(s) = s + 1.5 \times 10^{5}$$

Hence

$$y_{22a} = \frac{M(s)}{P(s)} = \frac{s^{2} + 3 \times 10^{5} s + 2 \times 10^{10}}{s + 1.5 \times 10^{5}}$$

and may be expanded as follows

$$y_{22a} = s + 1.33 \times 10^{5} + \frac{1.67 \times 10^{4} s}{s + 1.5 \times 10^{5}}$$

Then y_{22a} is separated as the input admittance of two parallel three-terminal networks.

$$y_{22a} = y_{22a}^{(1)} + y_{22a}^{(2)} = \left(s + \frac{8.35 \times 10^{3} s}{s + 1.5 \times 10^{5}} \right) + \left(1.33 \times 10^{5} + \frac{8.35 \times 10^{3} s}{s + 1.5 \times 10^{5}} \right)$$

Note that M(s) is also separated during this process as

$$M(s) = s (s + 1.58 \times 10^{5}) + (1.42 \times 10^{5} s + 2 \times 10^{10})$$

Polynomial D(s) can be separated as

$$D(s) = s (s + 1.3 \times 10^{3}) + (1.13 \times 10^{4} s + 1.58 \times 10^{10})$$

Therefore, the transfer functions of the two subnetworks may be written

$$T_{12a}^{(1)} = - \frac{s + 1.3 \times 10^{3}}{s + 1.58 \times 10^{4}}$$

$$T_{12a}^{(2)} = - \frac{1.13 \times 10^{4} s + 1.58 \times 10^{10}}{1.42 \times 10^{5} s + 2 \times 10^{10}}$$

The $y_{12a}^{(1)}$ and $y_{22a}^{(2)}$ are simplified by taking out the series capacitance and the series resistance

$$\frac{1}{y_{22a}^{(1)}} = \frac{1}{1.05 s} + \frac{1}{Y_{22a}^{(1)}}$$

$$\frac{1}{y_{22a}^{(2)}} = 7.05 \times 10^{-6} + \frac{1}{Y_{22a}^{(2)}}$$

where $Y_{22a}^{(1)} = 20s + 31.6 \times 10^{5}$ $\quad Y_{22a}^{(2)} = 15.8\,s + 1.22 \times 10^{6}$

The new transfer admittances are

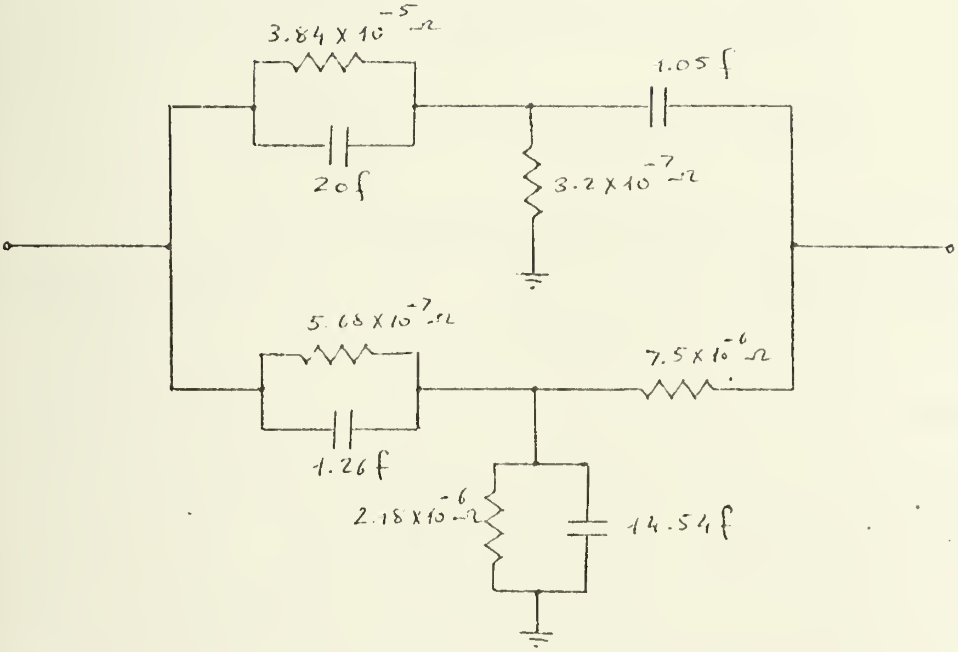
$$-Y_{12a}^{(1)} = T_{12a}^{(1)} \cdot Y_{22a}^{(1)} \quad , \quad -Y_{12a}^{(2)} = T_{12a}^{(2)} \cdot Y_{22a}^{(2)}$$

Hence

$$Y_{12a}^{(1)} = 20s + 2.6 \times 10^{4} \quad , \quad Y_{22a}^{(1)} = 20s + 2.6 \times 10^{4} + 31.34 \times 10^{5}$$

$$Y_{12a}^{(2)} = 1.26s + 1.76 \times 10^{6} \quad , \quad Y_{22a}^{(2)} = 1.26s + 1.76 \times 10^{6} + 14.54s + 4.6 \times 10^{5}$$

The feedback circuit is shown in Fig. IV.2.



$3.84 \times 10^{-5}\,\Omega$
$20\,f$
$1.05\,f$
$3.2 \times 10^{-7}\,\Omega$
$5.68 \times 10^{-7}\,\Omega$
$7.5 \times 10^{-6}\,\Omega$
$1.26\,f$
$2.18 \times 10^{-6}\,\Omega$
$14.54\,f$

Figure IV.2 The Feedback Circuit

The input circuit transfer function is

$$T_{12b}(s) = -\frac{1.15 \times 10^{4}\, s}{s^{2} + 3 \times 10^{5}\, s + 2 \times 10^{10}}$$

It has a simple zero at the origin and a simple zero at infinity. Therefore, it is not necessary to go through the

same procedure again. Instead by stating

$$y_{12b} = -\frac{N(s)}{P(s)} = -\frac{1.15 \times 10^4 s}{s + 1.5 \times 10^5}$$

$$y_{22b} = \frac{M(s)}{P(s)} = \frac{s^2 + 3 \times 10^5 s + 2 \times 10^{10}}{s + 1.5 \times 10^5} = s + 1.33 \times 10^5 + \frac{1.7 \times 10^4 s}{s + 1.5 \times 10^5}$$

The input circuit is shown in Fig. IV.3

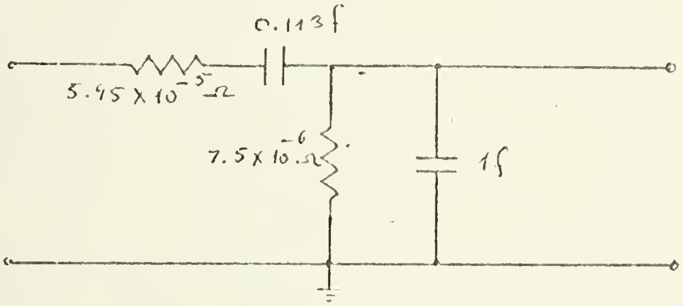


Figure IV.3 The Input Circuit

The overall circuit admittances are scaled by the scaling factor of 10^{-9}. The overall circuit diagram is shown in Fig. IV.4. The measured frequency response of the experimented circuit is given in table IV.1. The theoretical and the experimental frequency responses of the circuit are shown in Fig. IV.5. The theoretical and experimental curves are in very good agreement indicating that the circuit has good sensitivity with respect to both passive and active element variations.

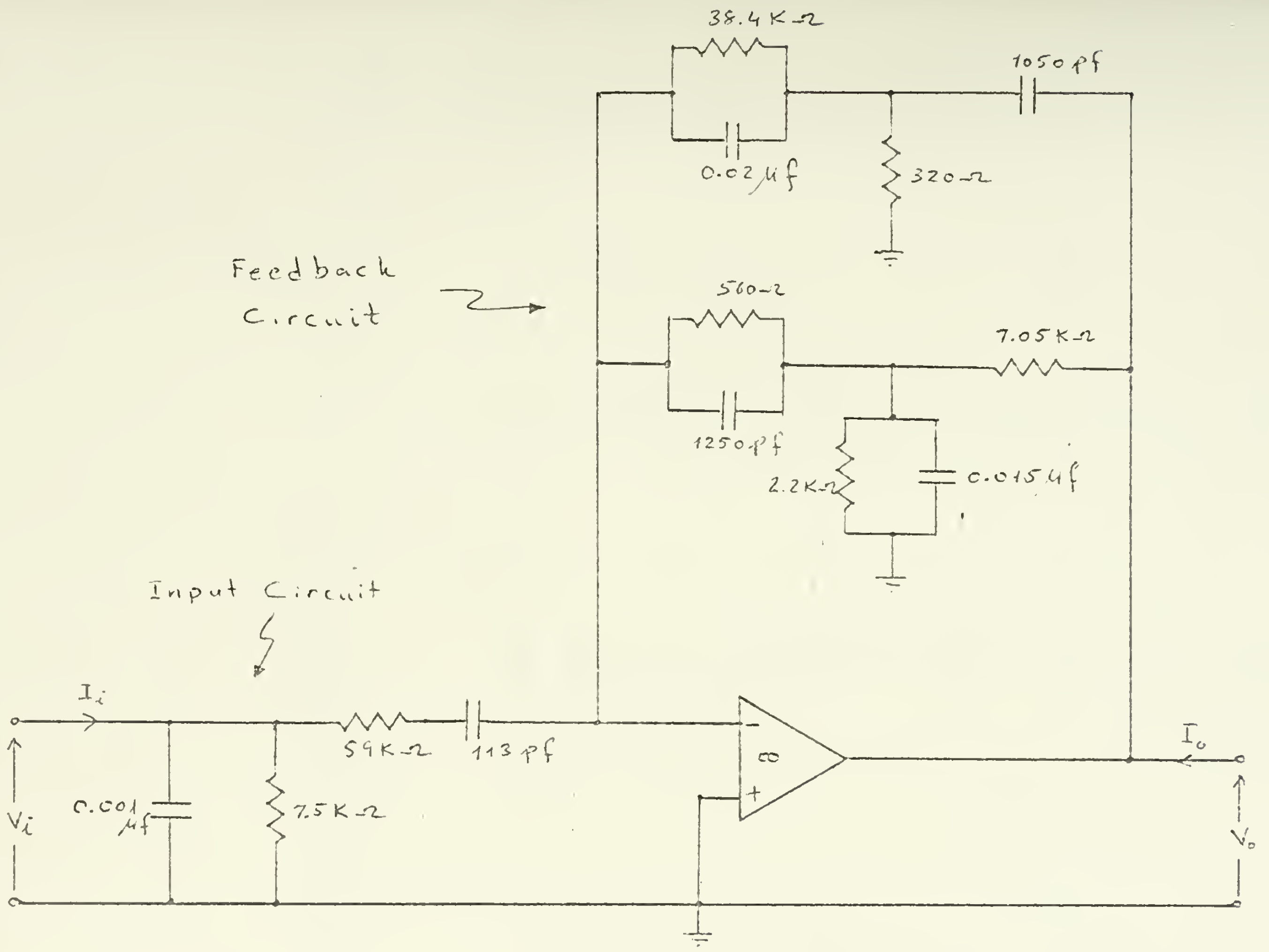


Figure IV.4 Overall Circuit Diagram of Example No. 1

FREQUENCY	GAIN	FREQUENCY	GAIN
10.0 Khz.	0.060	20.5 Khz.	0.900
11.0 "	0.074	21.0 "	0.700
12.0 "	0.090	21.5 "	0.600
13.0 "	0.115	22.0 "	0.440
14.0 "	0.130	22.5 "	0.300
15.0 "	0.150	23.0 "	0.267
16.0 "	0.200	24.0 "	0.230
17.0 "	0.267	25.0 "	0.193
17.5 "	0.293	26.0 "	0.160
18.0 "	0.380	27.0 "	0.134
18.5 "	0.500	28.0 "	0.120
19.0 "	0.700	29.0 "	0.110
19.5 "	0.900	30.0 "	0.100
20.0 "	1.000		

TABLE IV.1 The Experimental Frequency Response of Example No. 1.

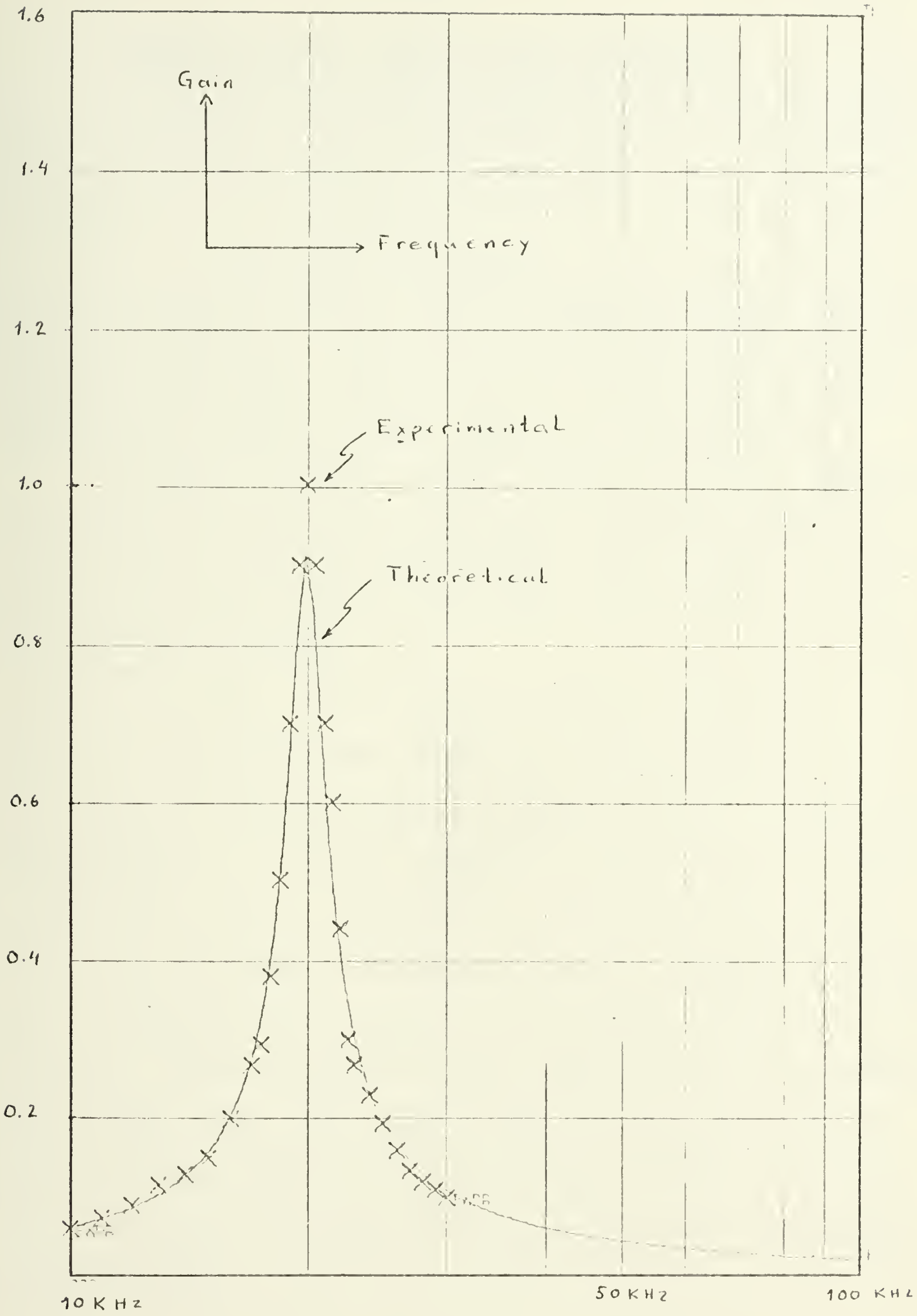


Figure IV.5 The Theoretical and the Experimental Frequency Responses of the Example No. 1

2. Example No. 2: Double-Ladder Synthesis

A second order high-pass open circuit voltage transfer function T(s), given by Huelsman [40]

$$T(s) = -\frac{s^2}{s^2 + 890s + 3.95 \times 10^5}$$

is to be realized with the double-ladder technique described in Section III.B.3b. The structure of Fig. III.8 and its accompanying circuit transfer function is considered.

In this particular example, both Y_2 and Y_4 have to be selected as capacitive admittances since the transfer function has two zeros at the origin. The element values were chosen as follows:

$$Y_1 = 296 \text{ mhos}, \quad Y_2 = Y_3 = Y_4 = s, \quad Y_{23} = 1370 \text{ mhos}$$

All of the admittances are scaled by a scaling factor of 10^{-8}. Then the circuit diagram becomes as shown in Fig. IV.6. The experimental frequency response of the circuit is given in table IV.2 and the experimental curve is given in Fig. IV.7. As can be seen a bandpass and not a highpass characteristic (as predicted by theory and also shown in Fig. IV.7) is obtained. The discrepancy at the higher end of the frequency band is due to the fact that the gain of the operational amplifier falls with frequency at a rate of 6db per octave and so progressively departs more and more from the assumed infinite value.

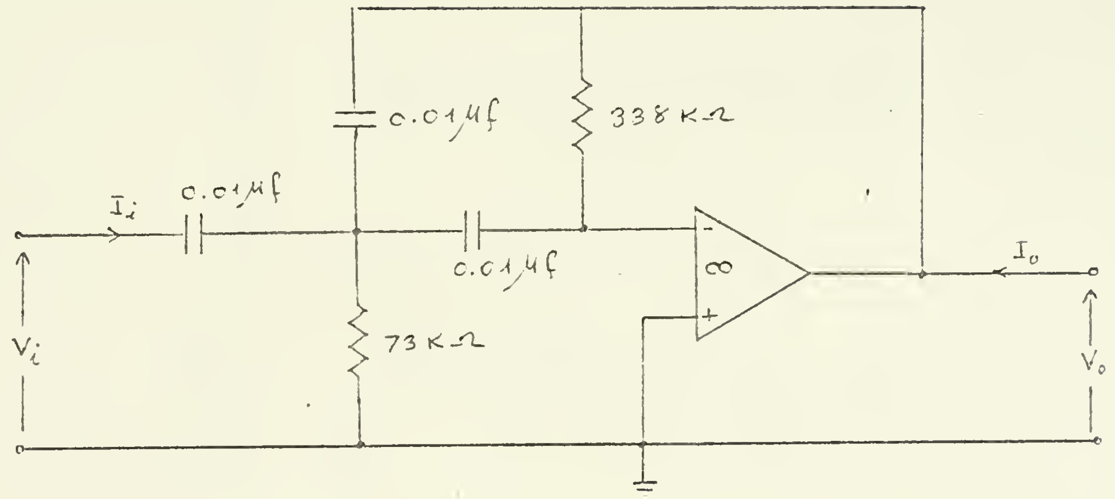


Figure IV.6 The Circuit Diagram of the Example No. 2.

FREQUENCY	GAIN	FREQUENCY	GAIN
1 Hz.	0.00	3	0.98
20 "	0.04	4	0.98
40 "	0.13	5	0.98
60 "	0.30	6	0.98
80 "	0.50	7	0.98
100 "	0.66	8	0.99
120 "	0.79	9	0.95
150 "	0.89	15	0.88
200 "	0.95	30	0.69
300 "	0.98	50	0.60
500 "	0.98	100	0.30
700 "	1.01	200	0.18
1 Khz.	0.98	500	0.14
2 Khz.	0.98	1 Mhz.	0.12

TABLE IV.2 The Experimental Frequency Response of the Example No. 2

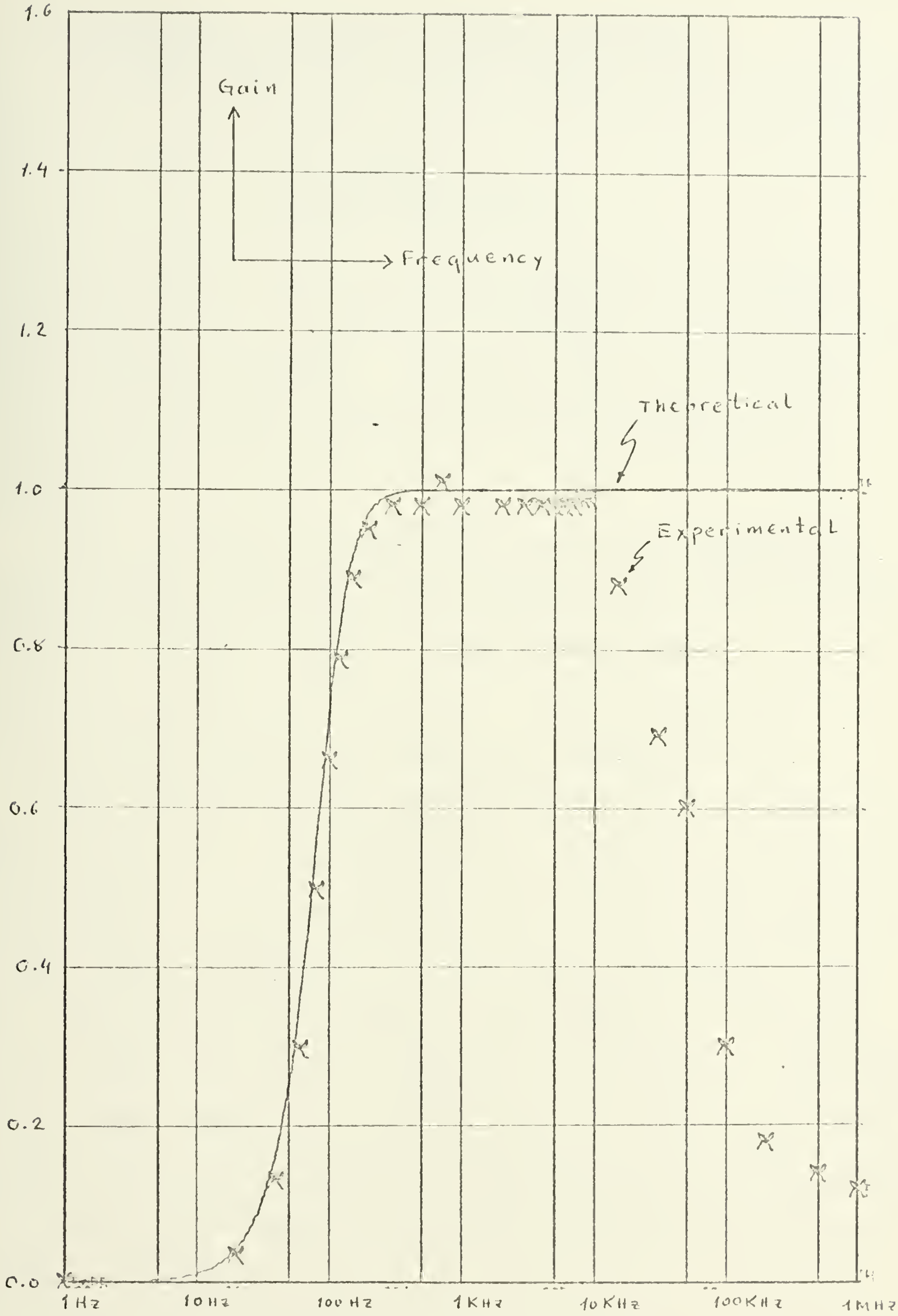


Figure IV.7 The Theoretical and the Experimental Frequency Responses of the Example No. 2.

3. Example No. 3: Single-Ladder Synthesis

A second order open circuit voltage transfer function T(s),

$$T(s) = -\frac{314 s}{s^2 + 0.0628 s + 9.9}$$

is to be realized. This transfer function has a bandpass characteristic with a low center frequency of 0.5 Hertz and a quality factor of fifty. The realization is to be made with the single-ladder synthesis technique described in Section III.B.3c.

The structure of Fig. III.13 and its accompanying circuit transfer function is considered. For this particular example the element values are chosen as follows

$Y_1 = 0.0314$ mhos , $Y_2 = Y_3 = s$, $Y_4 = 314$ mhos

All of the admittances are scaled by a factor of 10^{-6}.

The overall circuit configuration of this example is shown in Fig. IV.8. The experimental frequency response for this example is given in the table IV.3. The theoretical and the experimental responses of the circuit are shown in Fig. IV.9. Note the very high gain of the circuit at resonance. Note also that the feedback resistor of 32 Mohms is comparable with the input impedance of 2 Mohms for the operational amplifier and so it is not possible to assume that the input current of the amplifier is zero. This caused a slight shift in the resonant frequency.

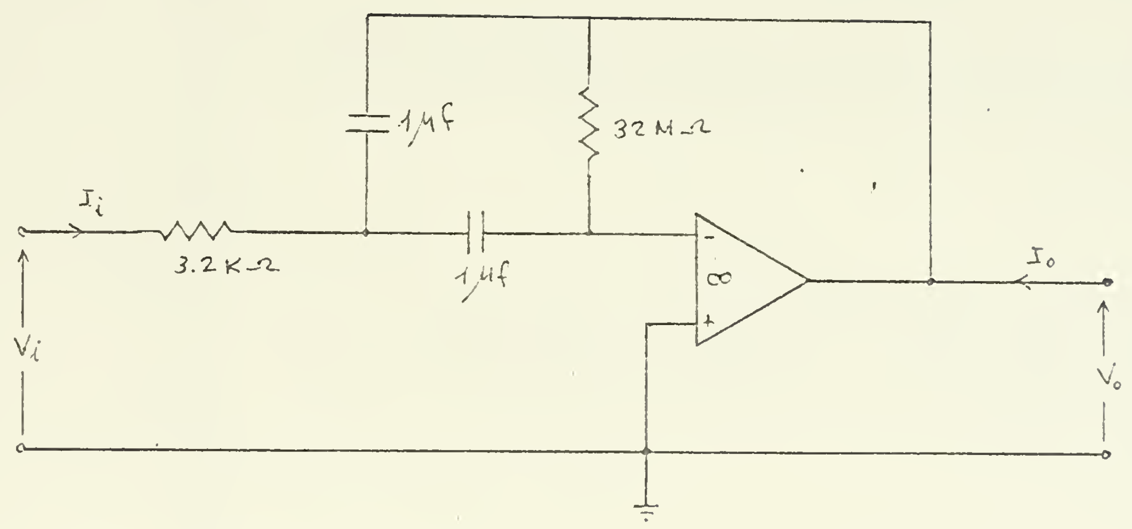


Figure IV.8 The Circuit Diagram of the Example No. 3.

FREQUENCY	GAIN	FREQUENCY	GAIN
0.100 Hz.	20	0.510 Hz.	1200
0.150 "	33	0.520 "	800
0.200 "	37	0.530 "	640
0.250 "	73	0.540 "	520
0.300 "	91	0.550 "	367
0.350 "	133	0.600 "	234
0.400 "	268	0.650 "	167
0.450 "	514	0.700 "	117
0.460 "	743	0.750 "	107
0.470 "	1183	0.800 "	94
0.480 "	2365	0.850 "	80
0.487 "	5000	0.900 "	67
0.490 "	3440	0.950 "	67
0.500 "	1800	1.000 "	60

TABLE IV.3 The Experimental Frequency Response of the Example No. 3

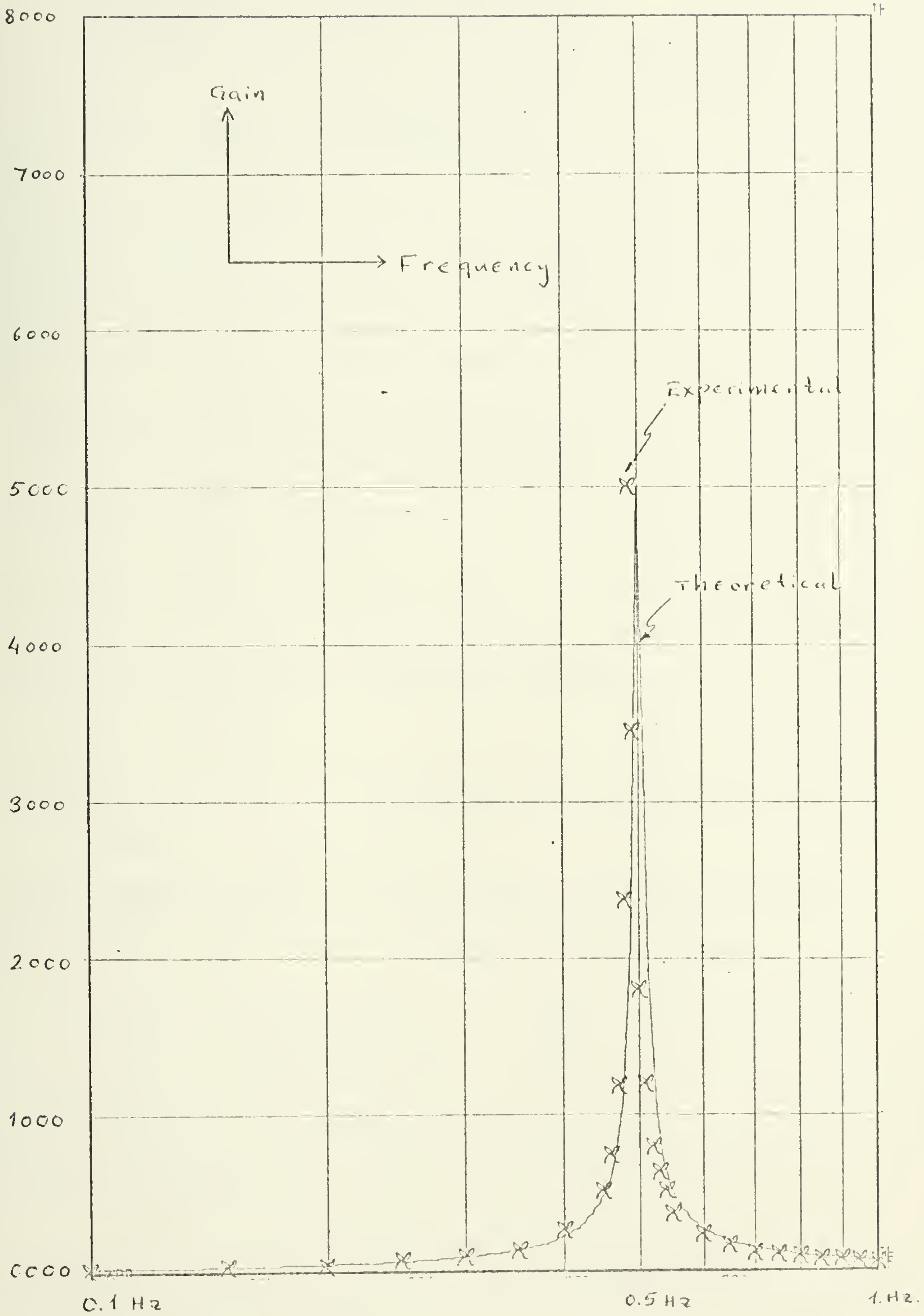


Figure IV.9 The Theoretical and the Experimental Frequency Responses of the Example No. 3

4. Example No. 4: Brugler's Circuit

A second order all-pass transfer function

$$T(s) = \frac{N(s)}{D(s)} = \frac{(s - 6.28 \times 10^3)^2}{(s + 6.28 \times 10^3)^2}$$

with a critical frequency in the 1 Khz. region is to be realized by Brugler's circuit described in Section III.B.3d. The general configuration of this circuit is shown in Fig. III.19. The voltage transfer function is

$$T(s) = \frac{Y_1(Y_4 + Y_5 + Y_6) - Y_4(Y_1 + Y_2 + Y_3)}{Y_6(Y_1 + Y_2 + Y_3) - Y_3(Y_4 + Y_5 + Y_6)} .$$

If in this transfer function

$$(Y_1 + Y_2 + Y_3) = (Y_4 + Y_5 + Y_6) \tag{IV.1}$$

then the voltage transfer function reduces to

$$T(s) = \frac{Y_1 - Y_4}{Y_6 - Y_3}$$

A polynomial P(s) is selected as previously described.

$$P(s) = s + 6.28 \times 10^3$$

Both the numerator and the denominator polynomials of the given transfer function is divided by P(s). Hence

$$\frac{N(s)}{P(s)} = s + 6.28 \times 10^3 - \frac{25.12 \times 10^3 s}{s + 6.28 \times 10^3} , \quad \frac{D(s)}{P(s)} = s + 6.28 \times 10^3$$

Then the values of the admittances, Y_1, Y_3, Y_4, Y_6 can be assigned as follows

$$Y_1 = Y_6 = s + 6.28 \times 10^3 , \quad Y_3 = 0 , \quad Y_4 = \frac{25.12 \times 10^3 s}{s + 6.28 \times 10^3}$$

In order to satisfy the Eq. IV.1 the values for Y_2 and Y_5 are assigned as

$$Y_2 = \frac{25.12 \times 10^3 s}{s + 6.28 \times 10^3} , \qquad Y_5 = 0$$

All of the admittances are scaled by a factor of 10^{-8}. The final circuit for this example is shown in Fig. IV.10. The experimental frequency response of this circuit is given in Table IV.4. The theoretical and the experimental frequency responses of the circuit are shown in Fig. IV.11. Note that the high frequency fall off of the experimental curve may be considered due to the fall off of the operational amplifier gain. The low frequency discrepancy at the theoretical and experimental curves is probably due to the poor sensitivity of the circuit, which is a characteristic common to all methcds which use the admittance difference terms in their denominator.

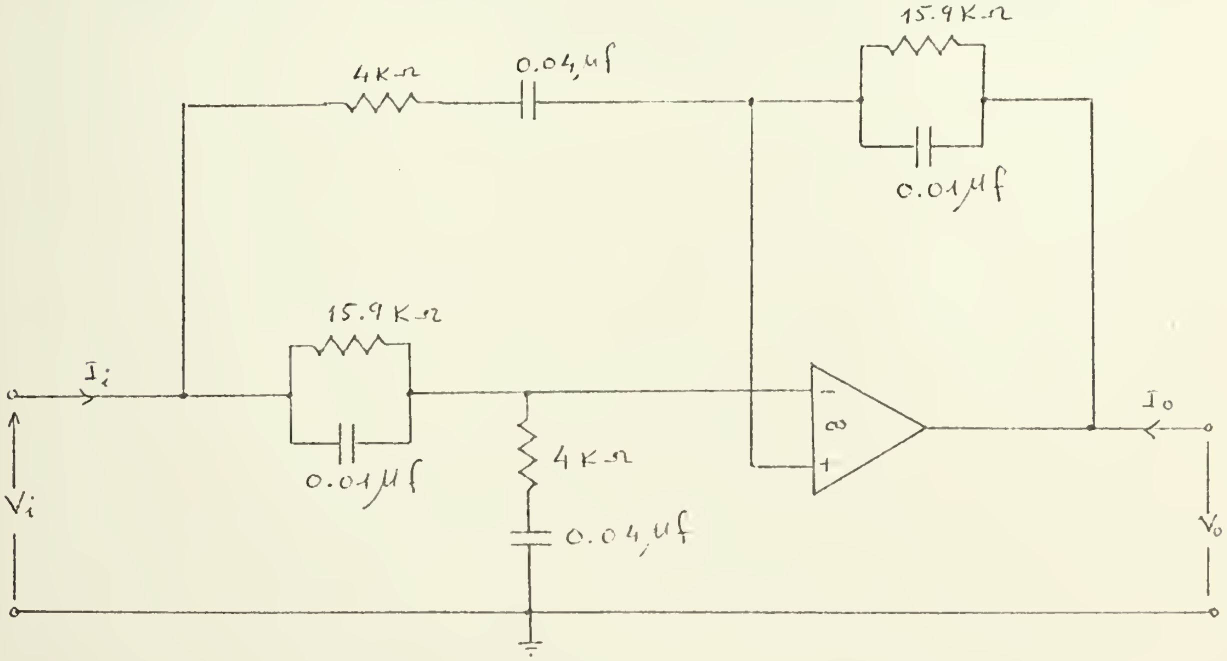


Figure IV.10. The Circuit Diagram of the Example No. 4.

FREQUENCY	GAIN	FREQUENCY	GAIN
10 Hz.	0.80	1.5 Khz.	0.98
50 "	0.85	2 "	0.98
100 "	0.88	3 "	1.00
250 "	0.92	5 "	1.00
500 "	0.95	10 "	1.00
700 "	0.98	20 "	1.01
800 "	0.98	50 "	1.02
900 "	0.98	100 "	1.03
1 Khz.	0.98	200 "	0.93
1.1 "	0.98	500 "	0.65
1.2 "	0.98	700 "	0.60
1.3 "	0.98	1 Mhz.	0.40

TABLE IV.4 The Experimental Frequency Response of the Example No. 4

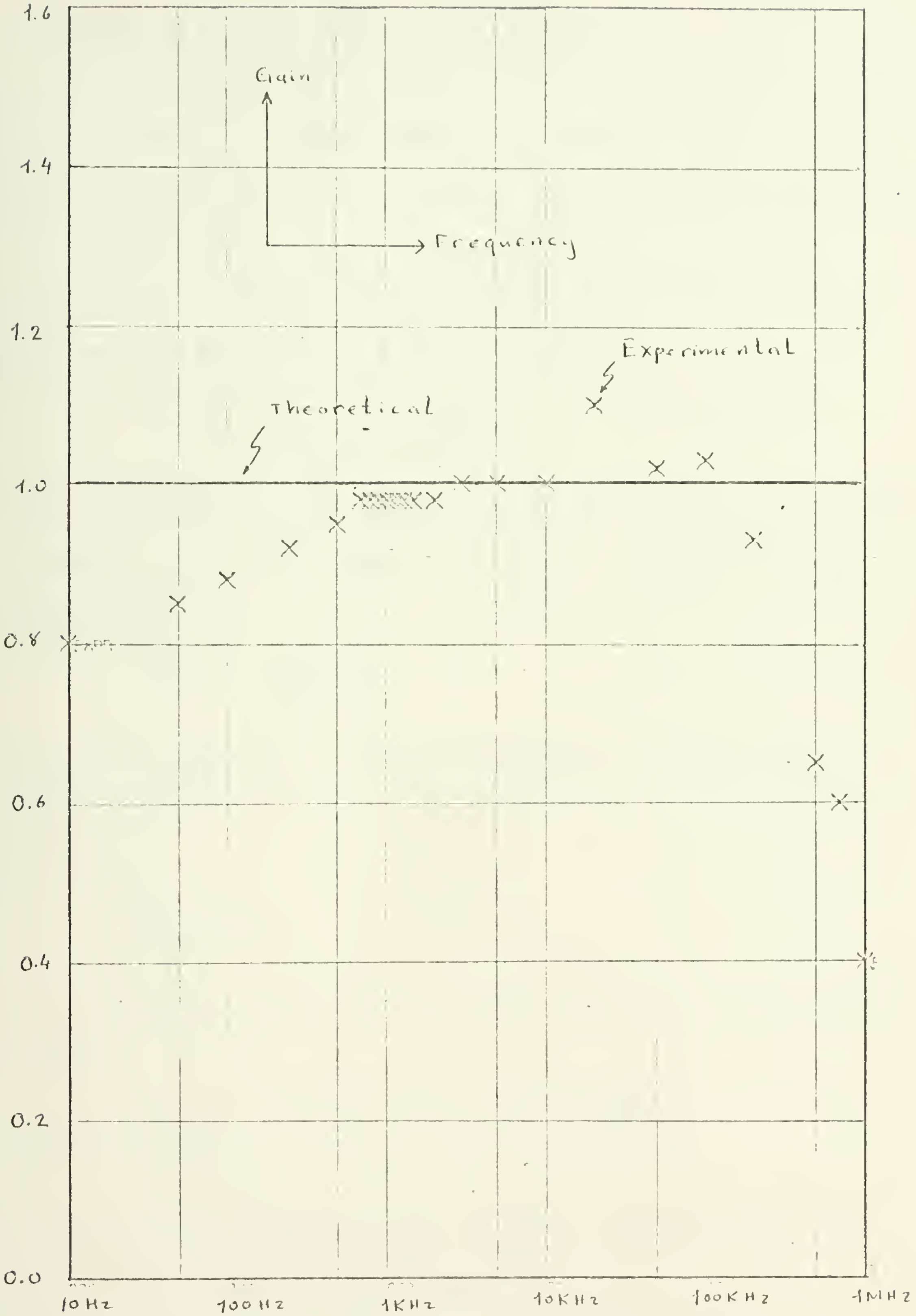


Figure IV.11 The Theoretical and the Experimental Frequency Responses of the Example No. 4

5. Example No. 5: Lovering's Circuit

The same all-pass voltage transfer function of example 4 is to be realized by the Lovering's circuit Fig. III.26, described in Section III.C.3. The given transfer function is

$$T(s) = \frac{N(s)}{D(s)} = \frac{(s - 6.28 \times 10^3)^2}{(s + 6.28 \times 10^3)^2}$$

The transfer function of the Fig. III.26 is

$$T(s) = \frac{Y_1 Y_5 - Y_2 Y_3}{Y_3 Y_6 - Y_4 Y_5}$$

If the admittances Y_3 and Y_5 are selected equal to each other but otherwise arbitrary, then the transfer function of the circuit reduces to

$$T(s) = \frac{Y_1 - Y_2}{Y_3 - Y_4}$$

Then both the numerator and the denominator polynomials of the given transfer function are divided by an auxiliary polynomial P(s),

$$P(s) = s + 6.28 \times 10^3$$

From that division, the values of the individual admittances are found.

$$Y_1 = Y_6 = s + 6.28 \times 10^3 , \quad Y_2 = \frac{25.12 \times 10^3 s}{s + 6.28 \times 10^3} , \quad Y_4 = 0$$

The values for the admittances Y_3 and Y_5 are selected arbitrarily as conductances of 10^5 mhos.

All admittances are scaled by a factor of 10^{-8}. The resulting circuit for this example is shown in Fig. IV.12. The experimental frequency response of the circuit is given

in Table IV.5. The theoretical and the experimental frequency responses of this example are shown in Fig. IV.13.

As can be seen, the circuit has good low frequency behavior. At high frequencies the characteristic falls off due to the fall off in gain of the operational amplifier. In addition, there is present some ringing probably due to over compensation by some parasitic elements.

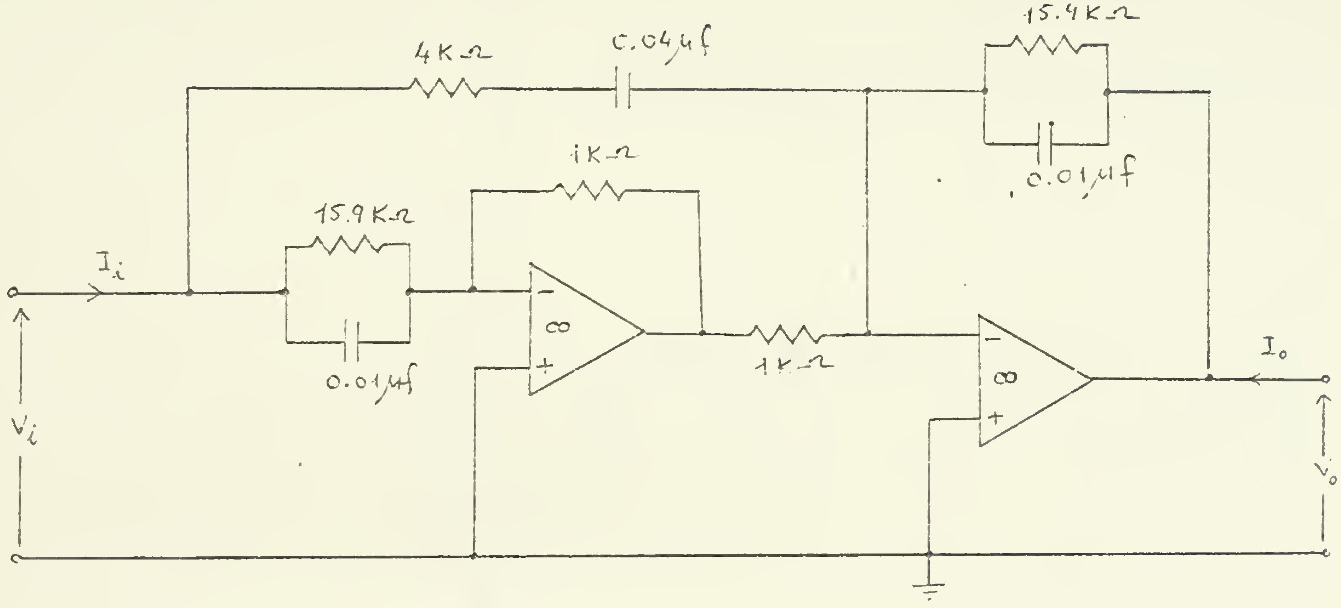


Figure IV.12   The Circuit Diagram of the Example No. 5

FREQUENCY	GAIN	FREQUENCY	GAIN
10 Hz.	1.00	1.5 Khz.	0.95
50 "	1.00	2 "	0.95
100 "	1.00	3 "	0.90
250 "	1.00	5 "	0.90
500 "	1.00	10 "	0.90
700 "	1.00	20 "	0.95
800 "	0.95	50 "	1.12
900 "	0.95	100 "	1.20
1 Khz.	0.95	200 "	1.00
1.1 "	0.95	500 "	0.70
1.2 "	0.95	700 "	0.65
1.3 "	0.95	1 Mhz.	0.70

TABLE IV.5 The Experimental Frequency Response of the Example No. 5

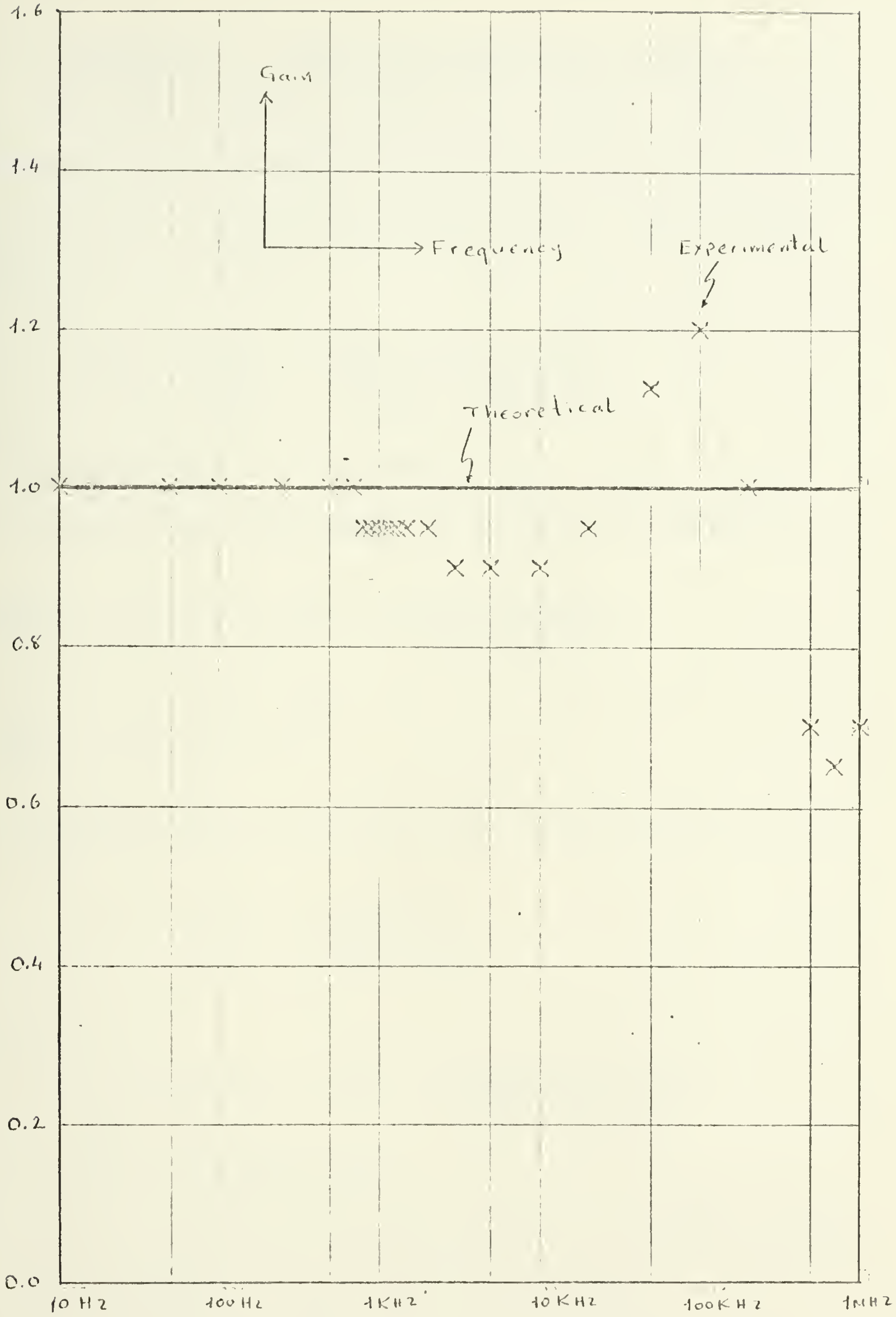


Figure IV.13 The Theoretical and the Experimental Frequency Responses of Example No. 5.

6. Example No. 6: Modified Lovering's Circuit

The same all-pass function of example 4 and 5 is to be realized by the modified Lovering's Circuit described in Section III.C.3, and shown in Fig. III.27. The transfer function is

$$T(s) = \frac{N(s)}{D(s)} = \frac{(s - 6.28 \times 10^3)^2}{(s + 6.28 \times 10^3)^2}$$

The same polynomial P(s) of example Nos. 4 and 5 is again used to divide both numerator and denominator polynomials.

$$\frac{N(s)}{P(s)} = s + 6.28 \times 10^3 - \frac{25.12 \times 10^3 s}{s + 6.28 \times 10^3}$$

$$\frac{D(s)}{P(s)} = s + 6.28 \times 10^3$$

The voltage transfer function of the circuit shown in Fig. III.27 is

$$T(s) = \frac{Y_b - Y_a}{Y_c - Y_b}$$

Therefore,

$$Y_b - Y_a = s + 6.28 \times 10^3 - \frac{25.12 \times 10^3 s}{s + 6.28 \times 10^3}$$

$$Y_c - Y_b = s + 6.28 \times 10^3$$

The values of individual admittances are $Y_a = \frac{25.12 \times 10^3 s}{s + 6.28 \times 10^3}$

$Y_b = s + 6.28 \times 10^3$, $Y_c = 2s + 12.56 \times 10^3$

By scaling the admittances with a factor of 10^{-8} the final circuit for this example becomes as shown in Fig. IV.14.

The experimental frequency response of this circuit is given in Table IV.6. The theoretical and the experimental frequency responses of this example are shown in Fig. IV.15. Again the circuit shows good low frequency behavior. The fall off at high frequency is due to the fall off in operational amplifier gain. However the circuit displays a considerable amount of ripple (or ringing) just before the fall off the characteristic and this is undesirable.

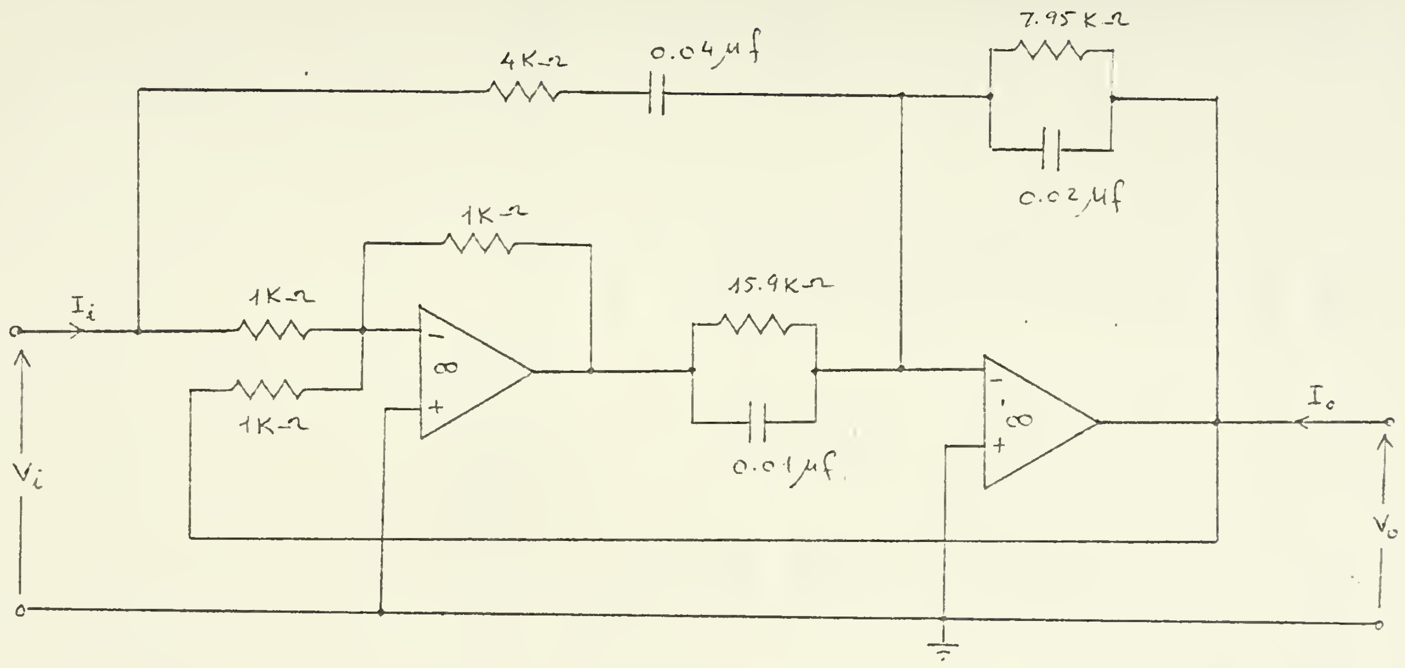


Figure IV.14 The Circuit Diagram of Example No. 6

FREQUENCY	GAIN	FREQUENCY	GAIN
10 Hz.	1.00	1.5 Khz.	0.90
50 "	1.00	2	0.95
100 "	1.00	3	1.00
250 "	1.00	5	1.05
500 "	0.96	10	0.90
700 "	0.93	20	1.10
800 "	0.90	50	0.90
900 "	0.90	100	0.40
1 Khz.	0.90	200	0.37
1.1 "	0.88	500	0.45
1.2 "	0.90	700	0.52
1.3 "	0.90	1 Mhz.	0.45

TABLE IV.6 The Experimental Frequency Response of the Example No. 6.

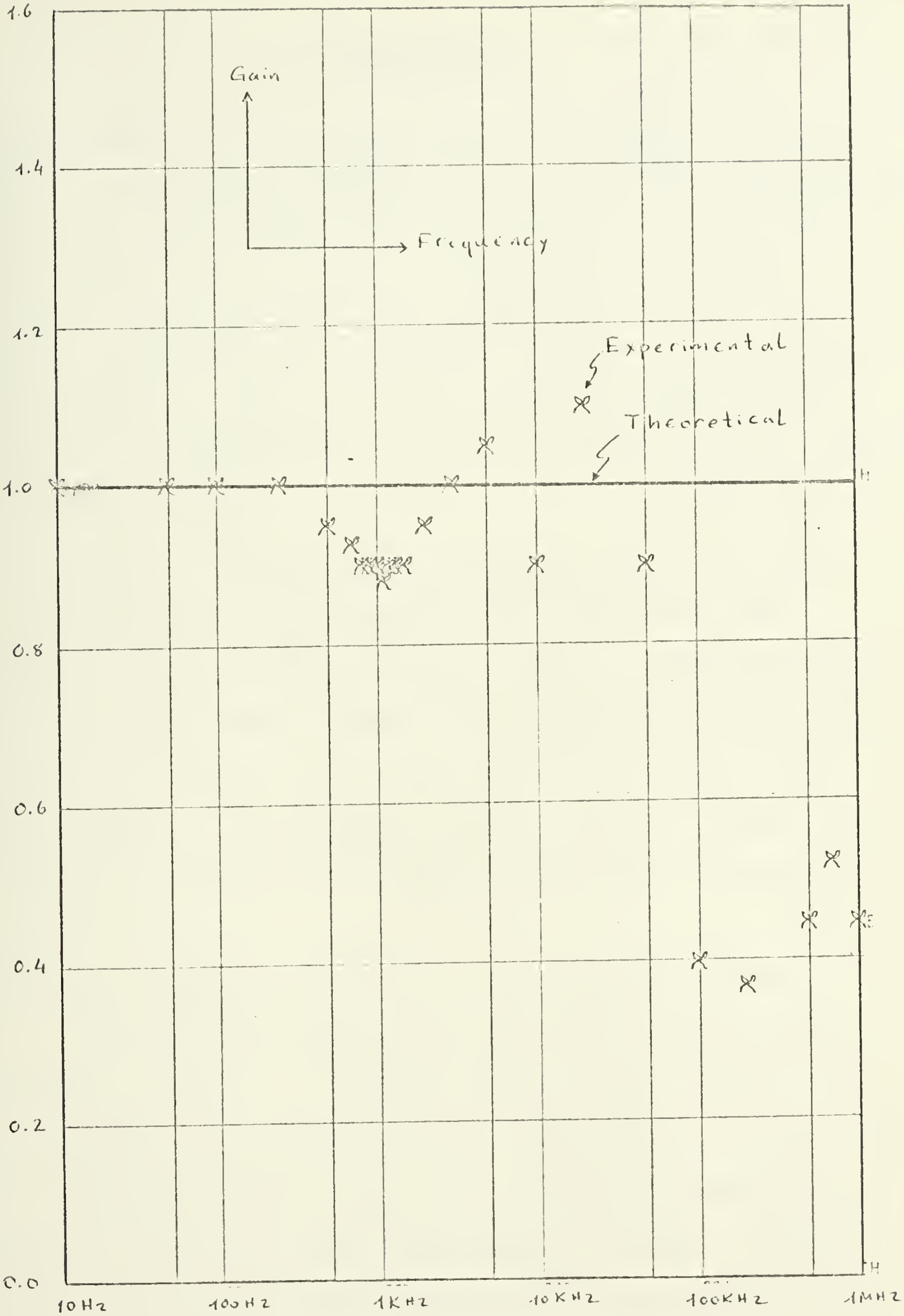


Figure IV.15 The Theoretical and the Experimental Frequency Responses of the Example No. 6.

V. CONCLUSIONS

Modern active network theory can be considered to have commenced in the middle nineteen fifties with the description of the first active RC filters. Since that time active RC synthesis has greatly expanded, particularly due to the introduction of practical integrated circuits in the middle nineteen sixties. There is every indication that active RC synthesis will completely revolutionize modern network theory in the immediate future.

This thesis is concerned in examining many of the circuits proposed in the last decade and classifying them according to their relative merits. This is accomplished by means of active RC theory whose main properties (often found scattered in the literature), are here collected and discussed in detail. In addition, several of the circuits described have been practically realized and their performance tested.

The main conclusions of this thesis are as follows:

1. Active RC network synthesis is a multiparameter optimization problem. In any practical circuit design one seeks a stable structure with least sensitivity to active and passive network parameter variations. One also attempts to minimize the number of active and passive circuit elements while yet ensuring that the element values required are suitable for integration.

2. Stability against oscillation is perhaps the most important factor, which must be considered. A careless design

can easily lead to an unstable, active RC structure and if so, makes pointless any further examination of sensitivity or other network performance criterion. Hence on the requirement of stability there can be no compromise with any other circuit design.

3. Perhaps the most useful figure of merit for a circuit is the sensitivity, which normally requires the solution of a multiparameter optimization problem. Various sensitivities such as gain, phase, Q, or pole sensitivities with respect to active and passive network parameters are distinguished and discussed. Two directions seem possible in order to optimize sensitivity. One is to design circuits with least sensitivity to parameter changes. The other is to design the circuits with moderate sensitivities, but reduce the magnitude of parameter variations. The first method is appropriate to passive elements, because of the wide tolerances usually encountered in integrated passive element realizations. The second method seems useful in the case of active elements, where the gain can be accurately controlled. If the desired network function can be made to depend on the ratio of passive network element values, then it is possible to keep variations within about one percent and so greater sensitivities may be allowed. Sensitivity studies complement stability studies and they are treated together in this thesis.

4. Integrated circuits have eliminated the need for designing with the least number of active elements. Because tolerances on passive elements are high, one attempts to

reduce their number and make critical network functions depend on ratios of element values, rather than on absolute element values. Use of capacitors is avoided if possible and resistance values must be kept within those limits, which can suitably be integrated. Very high resistances may be simulated by constant current sources and coupling capacitors may be replaced by level shifting circuits.

5. Certain circuits have high sensitivity to parameter variations. Among these circuits were Brugler's, Lovering's and Mathews-Seifert's circuit. Hence the extensive use of these circuits is not to be expected in the future. The single feedback synthesis does not suffer from high sensitivity, however the design procedure is complicated and tedious. The number of required passive elements is high and the error due to nonideal characteristics of operational amplifier cannot be predicted. Although sensitivities may be small, they are still complex functions of frequency and can in case of a careless design exhibit strange behavior. It seems that a simpler passive RC three terminal network synthesis is needed (other than Guillemin's or Fialkow-Gerst's technique), before the use of single feedback technique becomes practical. The double-ladder and single-ladder feedback techniques are advantageous, because the error of nonideal amplifiers can be predicted and corrected. Furthermore, the sensitivities are low and these techniques allow the circuit designer to select the most compact and useful element values. It is therefore felt that single and double-ladder synthesis techniques will

find extensive use in the future. One present drawback of these techniques is that as the degree of the desired transfer function becomes higher than the fourth degree, the calculation of element values become tedious and difficult. Further, the number of structures that may be selected becomes high and their relative merits are difficult to evaluate. State variable synthesis seems to have excellent properties because sensitivities are very low. Further, the error due to nonideal operational amplifiers is also low and can be neglected in many cases. Network functions depend on the ratio of resistors, which can be kept within about one percent tolerance. Finally, the circuit can be tuned by varying the input resistances of the summer amplifier and so the state variable synthesis technique will undoubtedly find extensive use in the future.

6. Final conclusions: It seems clear that passive inductors are becoming less and less important in network design and that active RC networks with operational amplifiers will obviate their need in the future. Naturally, the operational amplifier is not the only active device which may be used in a design. However it is presently the most versatile because of its low cost, high gain and ready availability. At present the cost of custom design of active RC networks for each required network function is deemed to be too high. The building of standard second-order blocks, which can be individually tuned and suitably cascaded, seems to hold much promise for the future. Nevertheless, there are many problems

that remain yet to be solved. Among these are the fact, that most of the presently available active filters have limited Q and that their performance is confined to quite low frequencies. Efforts in the future should be directed to increase the Q and the useful frequency range of these circuits besides decrease the sensitivities to parameter variations. Synthesis with distributed and a mixture of distributed and lumped circuits is also an appropriate subject for further study, because such structures may readily be integrated.

APPENDIX I

MATRIX ANALYSIS OF THE NETWORKS HAVING IDEAL OPERATIONAL AMPLIFIERS

An operational amplifier of gain A, having the input node i, and the output node j, in the linear, lumped, time invariant network imposes a constraint into the network by forcing the voltage V_j of the node j (with respect to the reference node), to follow that of the node voltage V_i in such a way that to maintain the relation:

$$V_j = AV_i \tag{1}$$

without, however, loading the node i.

Since an ideal operational amplifier has an infinite gain, $A=\infty$, and a finite output voltage in the closed loop form, then this imposes a virtual ground at the node i. Hence, $V_i = 0$. The operational amplifier injects a source current, I_j, into the node j, in such intensity as to enforce the virtual ground at the input node.

The admittance matrix of the network relates node voltages and source currents into the nodes, and is given in relation (2) below. Here i is the driving node, j is the driven node, p is the input node of the network, and q is the output node of the network.

Since the voltage at the driving node V_i is zero, it is possible to delete the i^{th} column from the admittance matrix and i^{th} element from the voltage vector simultaneously. Also since the value of the current imposed by the operational

$$\begin{bmatrix} Y_{11} & \cdot\cdot & Y_{1p} & \cdot\cdot & Y_{1i} & \cdot\cdot & Y_{1j} & \cdot\cdot & Y_{1q} & \cdot\cdot & Y_{1n} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{p1} & \cdot\cdot & Y_{pp} & \cdot\cdot & Y_{pi} & \cdot\cdot & Y_{pj} & \cdot\cdot & Y_{pq} & \cdot\cdot & Y_{pn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{i1} & \cdot\cdot & Y_{ip} & \cdot\cdot & Y_{ii} & \cdot\cdot & Y_{ij} & \cdot\cdot & Y_{iq} & \cdot\cdot & Y_{in} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{j1} & \cdot\cdot & Y_{jp} & \cdot\cdot & Y_{ji} & \cdot\cdot & Y_{jj} & \cdot\cdot & Y_{jq} & \cdot\cdot & Y_{jn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{q1} & \cdot\cdot & Y_{qp} & \cdot\cdot & Y_{qi} & \cdot\cdot & Y_{qj} & \cdot\cdot & Y_{qq} & \cdot\cdot & Y_{qn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdot\cdot & Y_{np} & \cdot\cdot & Y_{ni} & \cdot\cdot & Y_{nj} & \cdot\cdot & Y_{nq} & \cdot\cdot & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \\ V_p \\ \\ V_i \\ \\ V_j \\ \\ V_q \\ \\ V_n \end{bmatrix} = \begin{bmatrix} i_1 \\ \\ i_p \\ \\ i_i \\ \\ i_j + I_j \\ \\ i_q \\ \\ i_n \end{bmatrix} \tag{2}$$

amplifier, I_j is unknown it is better to delete the j^{th} row from the admittance matrix, and the j^{th} row from the current vector simultaneously. Then Eq. 2 can be rewritten as follows:

$$\begin{bmatrix} Y_{11} & \cdot\cdot & Y_{1p} & \cdot\cdot & Y_{1j} & \cdot\cdot & Y_{1q} & \cdot\cdot & Y_{1n} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{p1} & \cdot\cdot & Y_{pp} & \cdot\cdot & Y_{pj} & \cdot\cdot & Y_{pq} & \cdot\cdot & Y_{pn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{i1} & \cdot\cdot & Y_{ip} & \cdot\cdot & Y_{ij} & \cdot\cdot & Y_{iq} & \cdot\cdot & Y_{in} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{q1} & \cdot\cdot & Y_{qp} & \cdot\cdot & Y_{qj} & \cdot\cdot & Y_{qq} & \cdot\cdot & Y_{qn} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y_{n1} & \cdot\cdot & Y_{np} & \cdot\cdot & Y_{nj} & \cdot\cdot & Y_{nq} & \cdot\cdot & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ \vdots \\ V_p \\ \vdots \\ V_j \\ \vdots \\ V_q \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} i_1 \\ \vdots \\ i_p \\ \vdots \\ i_i \\ \vdots \\ i_q \\ \vdots \\ i_n \end{bmatrix} \tag{3}$$

The new admittance matrix of the Eq. 3 can be used to determine the system functions, provided that the input node p is not the driven node, and the output node q is not the driving node, that is:

$$p \neq j \text{ , and } q \neq i$$

which is satisfied in almost all cases.

In the current vector of the Eq. 3, the only non-zero entry is the input node current, i_p , which is the current injected by an input source. The node voltages of the network is found by pre-multiplying both sides of the Eq. 3 by the inverse of the admittance matrix.

$$\begin{bmatrix} V_1 \\ \vdots \\ V_p \\ \vdots \\ V_j \\ \vdots \\ V_q \\ \vdots \\ V_n \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} Y^{11} & \cdots & Y^{p1} & \cdots & Y^{i1} & \cdots & Y^{q1} & \cdots & Y^{n1} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y^{1p} & \cdots & Y^{pp} & \cdots & Y^{ip} & \cdots & Y^{qp} & \cdots & Y^{np} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y^{1j} & \cdots & Y^{pj} & \cdots & Y^{ij} & \cdots & Y^{qj} & \cdots & Y^{nj} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y^{1q} & \cdots & Y^{pq} & \cdots & Y^{iq} & \cdots & Y^{qq} & \cdots & Y^{nq} \\ \vdots & & \vdots & & \vdots & & \vdots & & \vdots \\ Y^{1n} & \cdots & Y^{pn} & \cdots & Y^{in} & \cdots & Y^{qn} & \cdots & Y^{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ i_p \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \qquad (4)$$

In Eq. 4, Δ denotes the determinant of the admittance matrix and the superscripts denote the cofactors.

The transfer function of the network is found from Eq. 4, as follows:

$$T(s) = \frac{V_q}{V_p} = \frac{Y^{pq} \cdot i_p}{Y^{pp} \cdot i_p} = \frac{Y^{pq}}{Y^{pp}} \qquad (5)$$

Without going through the proof presented above Eq. 5 can be used to determine the voltage transfer function of a network containing an operational amplifier.

The procedure is:

1. Find the admittance matrix of the passive part of the network.

2. Delete the row corresponding to the driven node, and the column corresponding to the driving node.

3. Provided that node p is not the driven node, and node q is not the driving node, the voltage transfer function of the active network is given by Eq. 5, where p is the input node, q is the output node, and the superscript denote the cofactors of the deleted admittance matrix.

The proof above is given for a single operational amplifier in the circuit, however the generalization of the procedure is immediate, with the constraint that the node p, is not one of the driven nodes, and the node q is not one of the driving nodes.

INTERNALLY COMPENSATED OPERATIONAL AMPLIFIER

FAIRCHILD LINEAR INTEGRATED CIRCUITS

FEATURES:

- NO FREQUENCY COMPENSATION REQUIRED
- SHORT-CIRCUIT PROTECTION
- OFFSET VOLTAGE NULL CAPABILITY
- LARGE COMMON-MODE AND DIFFERENTIAL VOLTAGE RANGES
- LOW POWER CONSUMPTION
- NO LATCH UP

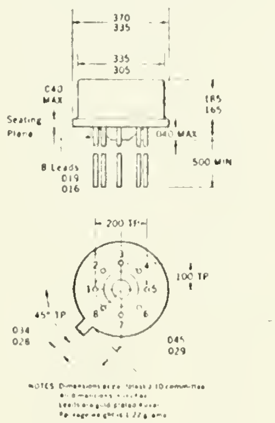
GENERAL DESCRIPTION — The µA741C is a high performance monolithic operational amplifier constructed on a single silicon chip, using the Fairchild Planar* epitaxial process. It is intended for a wide range of analog applications. High common mode voltage range and absence of "latch-up" tendencies make the µA741C ideal for use as a voltage follower. The high gain and wide range of operating voltages provide superior performance in integrator, summing amplifier, and general feedback applications. The µA741C is short circuit protected, has the same pin configuration as the popular µA709C operational amplifier, but requires no external components for frequency compensation. The internal 6dB/octave roll-off insures stability in closed loop applications. For full temperature range operation (−55°C to +125°C) see µA741 data sheet.

ABSOLUTE MAXIMUM RATINGS

Supply Voltage	±18 V
Internal Power Dissipation (Note 1)	500 mW
Differential Input Voltage	±30 V
Input Voltage (Note 2)	±15 V
Voltage between Offset Null and V−	±0.5 V
Storage Temperature Range	−65°C to +150°C
Operating Temperature Range	0°C to +70°C
Lead Temperature (Soldering, 60 sec)	300°C
Output Short-Circuit Duration (Note 3)	Indefinite

PHYSICAL DIMENSIONS

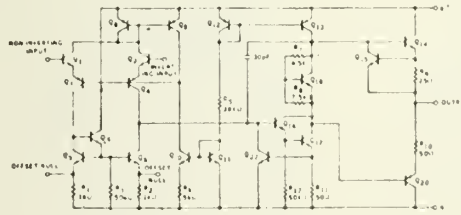
in accordance with
JEDEC (TO-99) outline



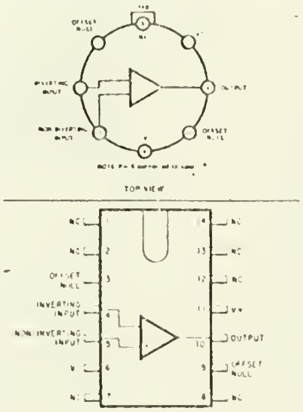
NOTES: Dimensions are in inches. Leads are gold-plated kovar. Package weight is 1.22 grams.

ORDER PART NO. U5B7741393

EQUIVALENT CIRCUIT

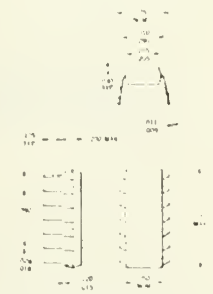


CONNECTION DIAGRAMS



TYPICAL DUAL IN-LINE PACKAGE

Similar to
JEDEC (TO-116) Outline



NOTES: All dimensions in inches
Leads are intended for insertion in hole rows .300 centers

ORDER PART NO. U6E7741393

NOTES:

(1) Rating applies for ambient temperatures to +70°C.
(2) For supply voltages less than ±15 V, the absolute maximum input voltage is equal to the supply voltage.
(3) Short circuit may be to ground or either supply.

*Planar is a patented Fairchild process.

FAIRCHILD [illegible]

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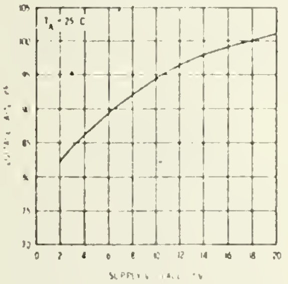
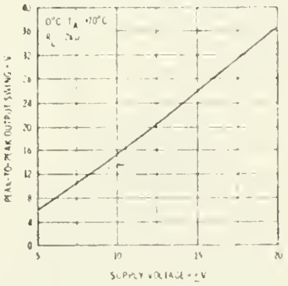
MANUFACTURED UNDER ONE OR MORE OF THE FOLLOWING U. S. PATENTS: 2981877, 3015048, 3025589, 3064167, 3108359, 3117260; OTHER PATENTS PENDING.

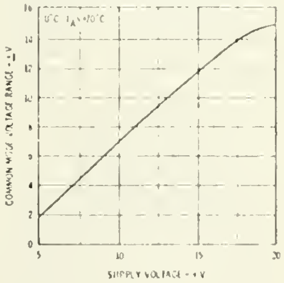
ELECTRICAL CHARACTERISTICS ($V_S = \pm 15$ V, $T_A = 25°C$ unless otherwise specified)

PARAMETERS (see definitions)	CONDITIONS	MIN.	TYP.	MAX	UNITS
Input Offset Voltage	$R_S \leq 10$ kΩ		2.0	6.0	mV
Input Offset Current			20	200	nA
Input Bias Current			80	500	nA
Input Resistance		0.3	2.0		MΩ
Input Capacitance			1.4		pF
Offset Voltage Adjustment Range			±15		mV
Input Voltage Range		±12	±13		V
Common Mode Rejection Ratio	$R_S \leq 10$ kΩ	70	90		dB
Supply Voltage Rejection Ratio	$R_S \leq 10$ kΩ		30	150	μV/V
Large Signal Voltage Gain	$R_L \geq 2$ kΩ, $V_{out} = \pm 10$ V	20,000	200,000		
Output Voltage Swing	$R_L \geq 10$ kΩ	±12	±14		V
	$R_L \geq 2$ kΩ	±10	±13		V
Output Resistance			75		Ω
Output Short-Circuit Current			25		mA
Supply Current			1.7	2.8	mA
Power Consumption			50	85	mW
Transient Response (unity gain)	$V_{in} = 20$ mV, $R_L = 2$ kΩ, $C_L \leq 100$ pF				
Risetime			0.3		μs
Overshoot			5.0		%
Slew Rate	$R_L \geq 2$ kΩ		0.5		V/μs
The following specifications apply for $0°C \leq T_A \leq +70°C$:					
Input Offset Voltage				7.5	mV
Input Offset Current				300	nA
Input Bias Current				800	nA
Large Signal Voltage Gain	$R_L \geq 2$ kΩ, $V_{out} = \pm 10$ V	15,000			
Output Voltage Swing	$R_L \geq 2$ kΩ	±10	±13		V

TYPICAL PERFORMANCE CURVES

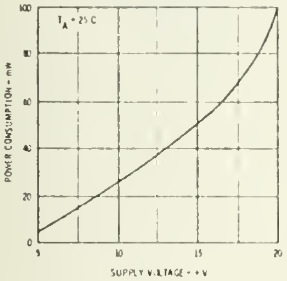

OPEN LOOP VOLTAGE GAIN AS A FUNCTION OF SUPPLY VOLTAGE


OUTPUT VOLTAGE SWING AS A FUNCTION OF SUPPLY VOLTAGE

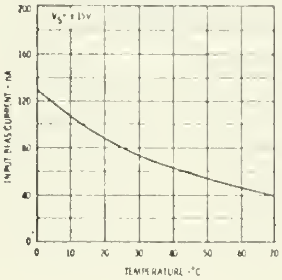

INPUT COMMON MODE VOLTAGE RANGE AS A FUNCTION OF SUPPLY VOLTAGE

TYPICAL PERFORMANCE CURVES

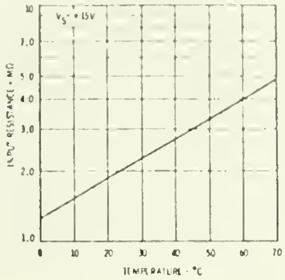
POWER CONSUMPTION AS A FUNCTION OF SUPPLY VOLTAGE



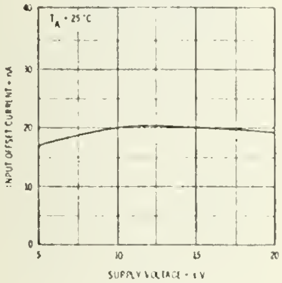
INPUT BIAS CURRENT AS A FUNCTION OF AMBIENT TEMPERATURE



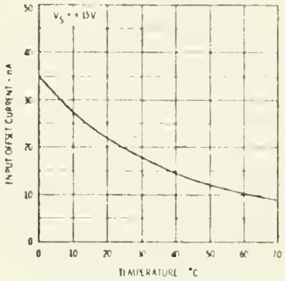
INPUT RESISTANCE AS A FUNCTION OF AMBIENT TEMPERATURE



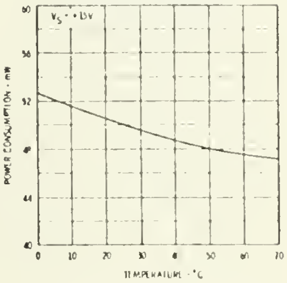
INPUT OFFSET CURRENT AS A FUNCTION OF SUPPLY VOLTAGE



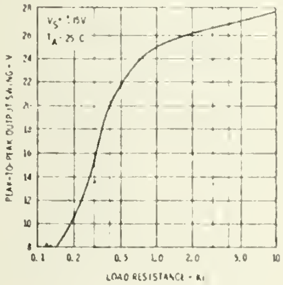
INPUT OFFSET CURRENT AS A FUNCTION OF AMBIENT TEMPERATURE



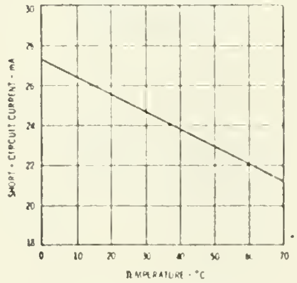
POWER CONSUMPTION AS A FUNCTION OF AMBIENT TEMPERATURE



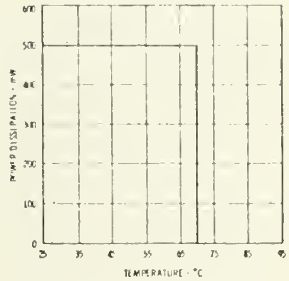
OUTPUT VOLTAGE SWING AS A FUNCTION OF LOAD RESISTANCE



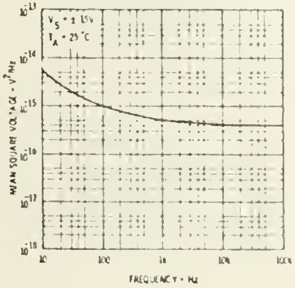
OUTPUT SHORT-CIRCUIT CURRENT AS A FUNCTION OF AMBIENT TEMPERATURE



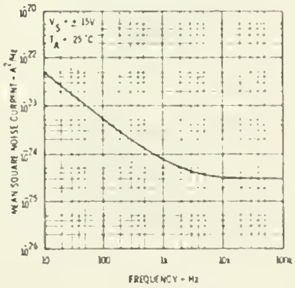
ABSOLUTE MAXIMUM POWER DISSIPATION AS A FUNCTION OF AMBIENT TEMPERATURE



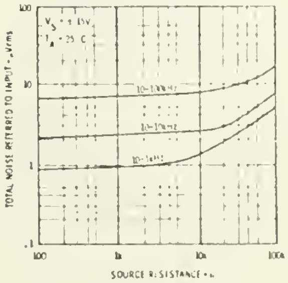
INPUT NOISE VOLTAGE AS A FUNCTION OF FREQUENCY



INPUT NOISE CURRENT AS A FUNCTION OF FREQUENCY

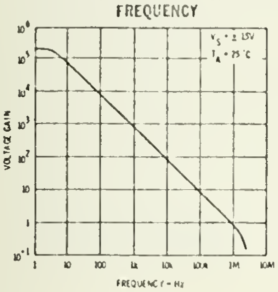


BROADBAND NOISE FOR VARIOUS BANDWIDTHS

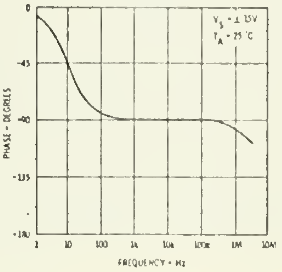


TYPICAL PERFORMANCE CURVES

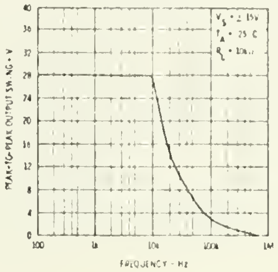
OPEN LOOP VOLTAGE GAIN AS A FUNCTION OF FREQUENCY



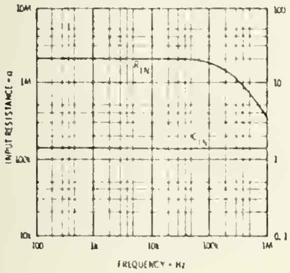
OPEN LOOP PHASE RESPONSE AS A FUNCTION OF FREQUENCY



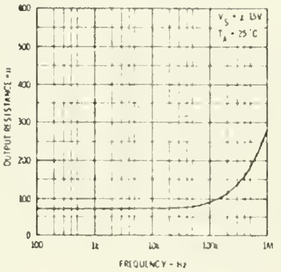
OUTPUT VOLTAGE SWING AS A FUNCTION OF FREQUENCY



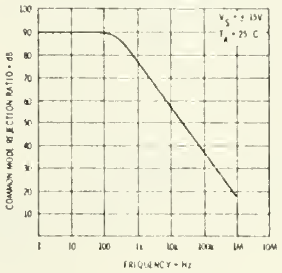
INPUT RESISTANCE AND INPUT CAPACITANCE AS A FUNCTION OF FREQUENCY



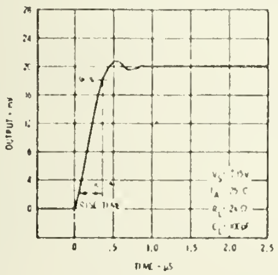
OUTPUT RESISTANCE AS A FUNCTION OF FREQUENCY



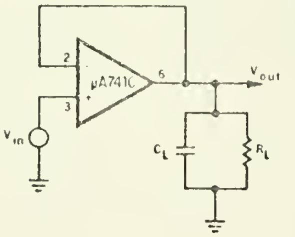
COMMON MODE REJECTION RATIO AS A FUNCTION OF FREQUENCY



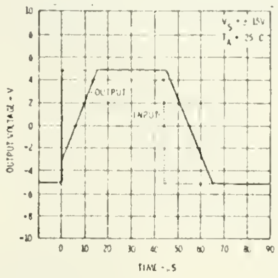
TRANSIENT RESPONSE



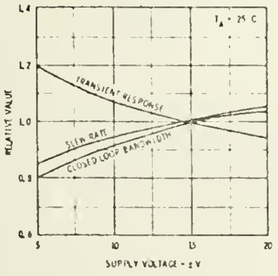
TRANSIENT RESPONSE TEST CIRCUIT



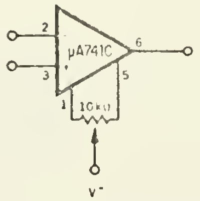
VOLTAGE FOLLOWER LARGE-SIGNAL PULSE RESPONSE



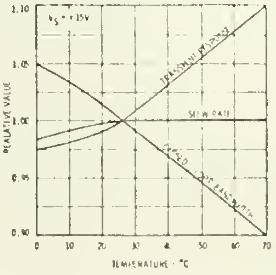
FREQUENCY CHARACTERISTICS AS A FUNCTION OF SUPPLY VOLTAGE



VOLTAGE OFFSET NULL CIRCUIT



FREQUENCY CHARACTERISTICS AS A FUNCTION OF AMBIENT TEMPERATURE



DEFINITION OF TERMS

INPUT OFFSET VOLTAGE — That voltage which must be applied between the input terminals to obtain zero output voltage. The input offset voltage may also be defined for the case where two equal resistances are inserted in series with the input leads.

INPUT OFFSET CURRENT — The difference in the currents into the two input terminals with the output at zero volts.

INPUT BIAS CURRENT — The average of the two input currents.

INPUT RESISTANCE — The resistance looking into either input terminal with the other grounded.

INPUT CAPACITANCE — The capacitance looking into either input terminal with the other grounded.

LARGE-SIGNAL VOLTAGE GAIN — The ratio of the maximum output voltage swing with load to the change in input voltage required to drive the output from zero to this voltage.

OUTPUT RESISTANCE — The resistance seen looking into the output terminal with the output at null. This parameter is defined only under small signal conditions at frequencies above a few hundred cycles to eliminate the influence of drift and thermal feedback.

OUTPUT SHORT-CIRCUIT CURRENT — The maximum output current available from the amplifier with the output shorted to ground or to either supply.

SUPPLY CURRENT — The DC current from the supplies required to operate the amplifier with the output at zero and with no load current.

POWER CONSUMPTION — The DC power required to operate the amplifier with the output at zero and with no load current.

TRANSIENT RESPONSE — The closed-loop step-function response of the amplifier under small-signal conditions.

INPUT VOLTAGE RANGE — The range of voltage which, if exceeded on either input terminal, could cause the amplifier to cease functioning properly.

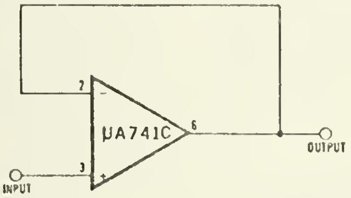
INPUT COMMON MODE REJECTION RATIO — The ratio of the input voltage range to the maximum change in input offset voltage over this range.

SUPPLY VOLTAGE REJECTION RATIO — The ratio of the change in input offset voltage to the change in supply voltage producing it.

OUTPUT VOLTAGE SWING — The peak output swing, referred to zero, that can be obtained without clipping.

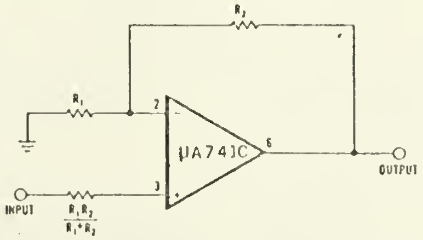
TYPICAL APPLICATIONS

UNITY-GAIN VOLTAGE FOLLOWER



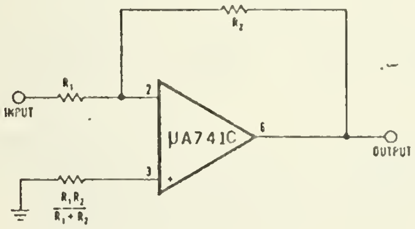
$R_{IN} = 400\ M\Omega$
$C_{IN} = 1\ pF$
$R_{out} << 1\ \Omega$
B.W. = 1 MHz

NON-INVERTING AMPLIFIER



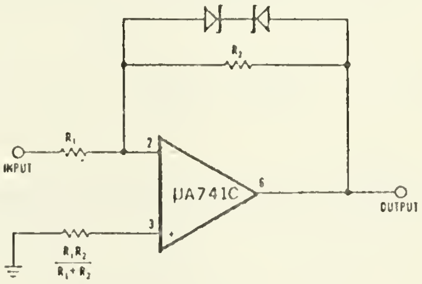
GAIN	R_1	R_2	B W	R_{IN}
10	1 kΩ	9 kΩ	100 kHz	400 MΩ
100	100 Ω	9.9 kΩ	10 kHz	280 MΩ
1000	100 Ω	99.9 kΩ	1 kHz	80 MΩ

INVERTING AMPLIFIER



GAIN	R_1	R_2	B W	R_{IN}
1	10 kΩ	10 kΩ	1 MHz	10 kΩ
10	1 kΩ	10 kΩ	100 kHz	1 kΩ
100	1 kΩ	100 kΩ	10 kHz	1 kΩ
1000	100 Ω	100 kΩ	1 kHz	100 Ω

CLIPPING AMPLIFIER



$$\frac{E_{out}}{E_{IN}} = \frac{R_2}{R_1} \text{ if } |E_{out}| \leq V_Z + 0.7\ V$$

where V_Z = Zener breakdown voltage

LIST OF REFERENCES

[1] Crisson, G., "Negative Impedances and the Twin-21-Type Repeater," Bell Sys. Tech. J., v. 10, pp. 485-513, July 1931.

[2] Scott, H. H., "A New Type of Selective Circuit and Some Applications," Proc. IRE, v. 26, pp. 226-235, February 1938.

[3] Bode, H. W., Network Analysis and Feedback Amplifier Design, Van Nostrand, N.Y. 1945.

[4] Phillips, Res. Rept., No. 3, The Gyrator; a New Network Element, by Tellegén, B.D.H., pp. 81-101, April 1948.

[5] Bogert, B. P., "Some Gyrator and Impedance Inverter Circuits," Proc. IRE, v. 43, pp. 793-796, July 1955.

[6] Sharpe, G. E., "Ideal Active Elements," J. Ins. Elec. Eng. (London), v. 3 (New Series), p. 431, July 1957.

[7] Shenoi, B. A., "Practical Realization of a Gyrator Circuit and RC-Gyrator Filters," IEEE Trans. on Circuit Theory, v. CT-12, No. 3, pp. 374-380, September 1965.

[8] Merill, J. L., Jr., "Theory of the Negative Impedance Converter," Bell Sys. Tech. J., v. 30, pp. 88-109, January 1951.

[9] Linvill, J. G., "Transistor Negative - Impedance Converters," Proc. IRE, v. 41, pp. 725-729, June 1953.

[10] Linvill, J. G., "RC Active Filters," Proc. IRE, v. 42, pp. 555-564, March 1954.

[11] Moschytz, G. S., "Miniaturized Filter Building Blocks Using Frequency Emphasizing Networks," Proc. National Electron Conference, pp. 364-367, 1967.

[12] Kerwin, W. J., Huelsman, L. P., and Newcomb, R. W., "State Variable Synthesis for Synthesis for Insensitive Integrated Circuit Transfer Functions," IEEE J. Solid State Circuits, v. SC-2, pp. 87-92, September 1967.

[13] Orchard, H. J., "Inductorless Filters," Electron. Letters, v. 2, p. 224, September 1966.

[14] Mitra, S. K., Analysis and Synthesis of Linear Active Networks, p. 106, John Wiley, N.Y., 1969.

[15] Bach, R. E., Jr., and Carlson, A. W., "Practical realization of Direct-Coupled Gyrator Circuits," *Proc. Asilomar Conf. on Circuits and Systems*, Monterey California, pp. 877-890, 1967.

[16] Kinariwala, B. K., "Necessary and Sufficient Conditions for the Existence of ± R, C Networks," *IRE - Trans. on Circuit Theory*, CT-7, pp. 330-335, September 1960.

[17] Lundry, W. R., "Negative Impedance Circuits: Some Basic Relations and Limitations," *IRE Trans. on Circuit Theory*, CT 4, pp. 132-139, September 1957.

[18] Mitra, S. K., "Equivalent Circuits of Gyrator," *Electron. Letters*, v. 3, pp. 333-334, July 1967.

[19] Vallese, L. M., "Understanding the Gyrator," *Proc. IRE*, v. 43, p. 483, April 1955.

[20] Larky, A. I., "Negative Impedance - Converters," *IRE Trans. on Circuit Theory*, v. CT-4, pp. 124-131, September 1957.

[21] Stanford Electron Labs. Report No. 6560-4, *RC Synthesis with Differential Input Operational Amplifiers, in Papers on Integrated Circuit Synthesis* (Compiled by Newcomb, R., and Rao, T.) Brugler, J.S., June 1966.

[22] Hakim, S. S., "RC Active Filters Using an Amplifier as the Active Element," *Proc. IEE (London)*, v. 112, pp. 901-912, May 1965.

[23] Van Valkenburg, M. E., *Introduction to Modern-Network Synthesis*, pp. 78-258, John Wiley, N. Y., 1962.

[24] Guillemin, E. A., "Synthesis of RC Networks," *J. Math. and Phys.*, v. 28, No. 1, April 1949.

[25] Fialkow, A., and Gerst, I., "The Transfer Function of General Two Terminal-Pair RC Networks," *Quarterly of App. Math.*, v. 10, pp. 113-127, April 1952.

[26] Hakim, S. S., "RC - Active Synthesis Using an Operational Amplifier," *Intern. J. of Contr.*, v. 1, 1st Series, No. 5, pp. 433-446, May 1965.

[27] Balabanian, N., and Cinkilie, T., "Expansion of an Active Synthesis Technique," *IEEE Trans. on Circuit Theory*, pp. 290-298, June 1963.

[28] Cinkilie, I., *Basic RC Network Structures for the Realization of Complex Transmission Zeros and Poles*, MS thesis, Syracuse University, Syracuse, N.Y., August 1962.

[29] Newell, W. E., "Selectivity and Sensitivity in Functional Blocks," *Proc. IRE*, v. 6, p. 2517, December 1962.

[30] Ur, H., "Root Locus Properties and Sensitivity Relations in Control Systems," *IRE Trans. on Auto. Cont.*, v. AC-5, pp. 57-65, January 1960.

[31] Kuo, F. F., "Pole-Zero Sensitivity in Network Functions," *IRE Trans. on Circuit Theory*, v. Ct-5, pp. 372-373, December 1958.

[32] Martinelli, G., "On the Matrix Analysis of Network Sensitivities," *Proc. IEEE*, v. 54, p. 72, January 1966.

[33] Sallen, R. P., and Key, E. L., "A Practical Method of Designing RC Active Filters," *IRE Trans. on Circuits Theory*, v. CT-2, pp. 74-85, March 1955.

[34] Horowitz, I. M., "Optimization of Negative Impedance Conversion Methods of Active RC Synthesis, "*Ire Trans. on Circuit Theory*, v. CT-6, No. 3, pp. 290-303, September 1959.

[35] Newcomb, R. W., *Active Integrated Circuit Design*, pp. 9-10, Prentice Hall, N.J., 1968.

[36] Schilling, D. L., and Belove, C., *Electronic Circuits: Discrete and Integrated*, pp. 335-339, McGraw-Hill, N.Y., 1968.

[37] Leeds, M. B., "Linear IC's: Inside the Operational Amplifier," *Electronics*, pp. 86-91, 16 October 1967.

[38] McVey, P. J., "Sensitivity in Some Simple RC Networks," *Proc. IEE (London)*, v. 112, pp. 1263-1269, July 1965.

[39] Gaash, A. A., Pepper, R. S., and Pederson, D. O., "Design of Integrable Desensitized Frequency Selective Amplifiers," *ISSCC Dig. Tech. Papers*, p. 34, February 1966.

[40] Huelsman, L. P., *Theory and Design of Active RC Circuits*, pp. 231-272, McGraw Hill, N.Y., 1968.

[41] McVey, P. J., "Synthesis of Transfer Functions by RC Networks with Two or Three Computing Amplifiers," *Intern. J. Contr.*, v. 2, 1st Series, No. 2, pp. 125-134, August 1965.

[42] Deliyanis, T., "Sensitivity Study of Five RC - Active Networks Using the Method of Single Inversion," *Intern. J. Electronics*, v. 22, No. 3, pp. 197-213, March 1967.

[43] Bohn, E. V., Transfer Analysis of Linear Systems, p. 71, Addison-Wesley, Mass., 1963.

[44] Bridgeman, A., Brennan, R., "Simulation of Transfer Function Using only One Operational Amplifier," Proc. WESCON Convention Rec. 1, pt. 4, pp. 273-278, 1957.

[45] Wadhwa, L. K., "Simulation of Third-Order Systems with Double-Lead Using One Operational Amplifier," Proc. IRE, pp. 1518-1539, June 1962.

[46] Nichols, M. H., and Rauch, L., Radio Telemetry, p. 396, John-Wiley, N.Y., 1956.

[47] Hold, A. G. J. and Sewell, J. I., "Table for the Voltage Transfer Functions of Single Amplifier Double-Ladder Feedback Systems," Electron. Letters, v. 1, No. 3, p. 70-71, May 1965.

[48] Nathan, A., "Matrix Analysis of Networks Having Infinite Gain Operational Amplifiers," Proc. IRE, p. 1577-1578, October 1961.

[49] Nathan, A., "Matrix Analysis of Constrained Networks," IEE (London) Monograph No. 399,E, September 1960.

[50] Aggarwal, G. K., "A Single Operational Amplifier Simulates Third-Order Systems," Proc. Indian. Acad. Sci., v. 58, p. 257, November 1963.

[51] Aggarwal, G. K., "On Fourth Order Simulation by One Amplifier," J. Electronics and Control, v. 15, p. 449, November 1963.

[52] Aggarwal, G. K., "A z-y Table for Analyzing a Ladder Network," Proc. IEE, v. 111, March 1964.

[53] Aggarwal, G. K., "On the n^{th} Order Simulation by One Operational Amplifier," Proc. IEEE, v. 52, No. 8, p. 969, August 1964.

[54] Aggarwal, G. K., "On Fifth Order Simulation," J. Inst. Tele., Engineers India, September 1964.

[55] Aggarwal, G. K., "On Using One Operational Amplifier for n^{th} Order Simulation," Intern. J. Control, v. 1, 1st Series, No. 6, pp. 557-564, June 1965.

[56] Hold, A. G. J., and Sewell, J. I., "Active RC Filters Employing a Single Operational Amplifier to Obtain Biquadratic Responses," Proc. IEE, v. 112, No. 12, pp. 2227-2235, December 1965.

[57] Mathews, M. V., and Seifert, W. W., "Transfer Function Synthesis with Computer Amplifiers and Passive Networks," Proc. Western Joint Computer Conference, pp. 7-12, March 1955.

[58] Lovering, W. F., "Analog Computer Simulation of Transfer Function," Proc. IEEE, v. 53, p. 306, March 1965.

[59] Zverev, A. I., Handbook of Filter Synthesis, p. 20, John Wiley, N.Y., 1967.

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ABSTRACT

Active RC network synthesis techniques with operational amplifiers are reviewed, discussed, and classified according to the number of amplifiers and number of feedback paths in the circuit. After presenting the main properties of active RC network theory, various synthesis techniques are discussed and evaluated according to their merits, by means of sensitivity and stability theory. A modification to Lovering's circuit is proposed.

Six design examples are presented to illustrate the application of the techniques and to observe the effect of nonideal active and passive components. The designs are practically realized, their performance is tested and experimental results are presented. Reasons for the discrepancies found between theory and experimental results are discussed.

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